Logics of agency

Chapter 4: Resource sensitive agency

Daniele Porello and Nicolas Troquard

KRDB Research Center for Knowledge and Data
Free University of Bozen-Bolzano

name.surname@unibz.it

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- We introduce resource sensitive logics, in particular based on *linear logic*.
- We propose a resource sensitive version of the bringing it about logic BIAT (based on the four principles that we discussed). [PT14, PT15]
- We show a few examples of modelling resource sensitive actions.
- We approach order-sensitivity: sequences of actions, priority, etc.
- Conclusions.

Resource-sensitive agency:

- Actions require resources to be performed,
- e.g. buying/selling an item, moving in an environment, transforming an environment, using a tool, etc.
- In fact, any action in principle requires spending resources to achieve something.
- In case we need to model explicitly the resources used to perform the action, one strategy is to replace classical logic with resource sensitive logics.

- Specifying "hammering": if I place a nail (N) and I provide the right force (F), then I can drive a nail (D) with the hammer.

  So assume:

$$\vdash N \wedge F \rightarrow D$$

- In classical logic: we cannot say that one hammering action is about *one* nail and *one* hammer, that let me drive in *one* nail:

$$\frac{\vdash N \wedge F \rightarrow D}{\vdash N \wedge F \rightarrow D \wedge \ldots \wedge D}$$

- Why?

Sequent calculi were introduced by Gentzen, 1937 to study the proof theory of logics (in particular, of classical and intuitionistic logic).

A sequent is an expression of the form:

$$\Gamma \vdash \Delta$$

Where Γ are the hypotheses of the sequent and Δ are the conclusions.

The intuitive meaning of the sequent is:

$$\gamma_1 \text{ and } \gamma_2 \cdots \text{ and } \gamma_m \;\; \textit{entail} \;\; \delta_1 \text{ or } \delta_2 \text{ or } \cdots \text{ or } \delta_m$$

A logic is then defined by means of two types of rules:

- *Structural Rules*: determine the structure of Γ and Δ (e.g. a set, a multiset, a list, a tree, etc.)
- *Logical Rules*: determine the behaviour of logical connectives (e.g. they distinguish the conjunction from the disjunction).

The *structural rules* of *contraction*, *weakening*, and *exchange* define how to deal with formulas in a proof:

$$\frac{\Gamma, A, A, \vdash \Delta}{\Gamma, A \vdash \Delta}\,(C) \qquad \frac{\Gamma \vdash \Delta}{\Gamma, A \vdash \Delta}\,(W) \qquad \frac{\Gamma, A, B, \Gamma' \vdash \Delta}{\Gamma, B, A, \Gamma' \vdash \Delta}\,(E)$$

I.e. W, C, and E entail that Γ and Δ are *sets*.

W, C, and E determine also the behaviour of logical connectives (Girard, 1987).

In particular they make the following two presentations of logical rules are equivalent:

$$\frac{\Gamma \vdash A \qquad \Delta \vdash B}{\Gamma, \Delta \vdash A \wedge B}\,\wedge \qquad \frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \wedge B}\,\wedge$$

(*multiplicative and additive presentation*)

- By rejecting structural rules, we are lead to define two conjunctions with different behaviour: multiplying contexts ($\otimes$) or identifying them ($\&$).

Unrestricted use of W and C is problematic for modelling the agency:

- Identifying tokens of actions (Contraction)

  E.g.: $\vdash N \to N \wedge N$

  If I place a nail one, then I place it twice

- Introducing arbitrary actions (Weakening)

  E.g.: $\vdash N \wedge C \to N$

  If I can place a nail, I can place it in any case (regardless of the context C)

- Insensitivity to the order of action (Exchange)

  E.g.: $\vdash N \wedge F \to F \wedge N$

  If I can place a nail and then apply a force, I can apply a force and then place a nail.

- Instantiating classical propositional reasoning in terms of actions leads to unintuitive outcomes.

An example from Girard 1995

Consider a petrol engine, in which petrol P causes the motion M:

$$P \vdash M$$

Weakening would enable any motion to be caused by a petrol engine:

$$\frac{\vdash M}{P \vdash M}\,(\mathrm{W})$$

Contraction makes miracles, free motion:

$$\frac{\dfrac{P \vdash P \qquad P \vdash M}{P, P \vdash P \wedge M}}{P \vdash P \wedge M}\,(\mathrm{C})$$

Basic observations

Need for *weak propositional logics* (substructural logics, i.e. control Weakening and Contraction) to handle resource-sensitive reasoning, e.g.:

- Bunched Implication (O'Hearn, Pym 1999);
- Relevant Logics (Anderson et Al, 1992)
- Linear Logic (Girard 1987).

Modal logics are the preferred tools to deal with agency and action, and more generally with propositional attitudes, e.g.:

- Group beliefs (Lismont, Mongin 1994, Porello 2018);
- Coalitional powers (Pauly 2002);
- Actual agency (Kanger, Belnap, and others).

We will combine both:
Linear Logic with *weak* (non-normal) *modalities*.

We propose:

- Minimal modal Linear logic:
  - Semantics
  - Hilbert-style axiomatisation
  - Sequent calculus
  - Cut-elimination
  - PSPACE membership of proof-search.
- Modal Linear Logic of Agency: extend the minimal modal linear logic by providing a resource-sensitive version of the core principles of agency.
- Case study: Resource-sensitive agency instantiated by the use of *artefacts* (hammers, screwdrivers, ..., web services, ...)

Our propositional language

A fragment of (Intuitionistic) Linear Logic, $\mathcal{L}_{\mathsf{ILL}}$, defined by the BNF

$$A ::= \mathbf{1} \mid p \mid A \otimes A \mid A \,\&\, A \mid A \multimap A$$

where $p \in Atom$.

$A \otimes B$: A and B (“tensor”; multiplicative (or intensional) conjunction)
$A \,\&\, B$: A and B (“with”; additive (or extensional) conjunction)
$A \multimap B$: A implies B (“lollipop”; linear implication)

Let $\bot \in Atom$ a designated atom to mean contradiction.

Negation defined: $\sim A \equiv A \multimap \bot$.

Other connectives in full Linear Logic: $⅋$; $\oplus$; $!$; $?$; $\mathbf{0}$, $\top$.

The meaning of linear logic connectives: A menu

- Price: 27 euros,
- Appetiser: Prosciutto e melone/fichi (depending on season)
- Primo: Spaghetti/Gnocchi,
- Drink: Water (as much as you like)

$$Pz \multimap ((P \otimes M) \oplus (P \otimes F)) \otimes (S \,\&\, G) \otimes !W$$

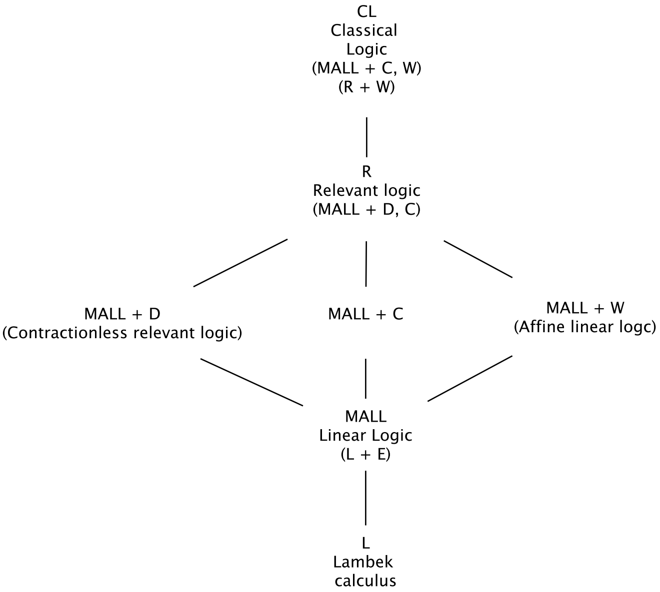
- $A \multimap B$: consuming one A, you get one B;
- $A \otimes B$: you have one copy of A and one of B.
  E.g. $A \otimes B \nvdash A$: in order to sell A and B, we need someone who buys both A and B.
- $A \oplus B$: you have one of the two but you cannot chose;
- $A \,\&\, B$: you have one of the two and you can chose;
- $!A$: use A *ad libitum* (! reintroduce structural rules)

Models for ILL: formally

A *Kripke resource model* [Urquhart 1972] is a structure $\mathcal{M} = (M, e, \circ, \prec, V)$, where:

- $(M, e, \circ)$ is a commutative monoid:
  - M is a set of resources;
  - $\circ$ is associative and commutative;
  - $m \circ e = m$.
- $\prec$ is a preorder (reflexive, transitive) on M;
- $V : Atom \to \mathcal{P}(M)$;
- if $m \prec n$, and $m' \prec n'$, then $m \circ m' \prec n \circ n'$ (bifunctoriality);
- if $m \in V(p)$ and $n \prec m$ then $n \in V(p)$ (heredity).

Hierarchy of logics wrt. resource-sensitivity



MILL: A MINIMAL MODAL EXTENSION OF ILL

The language of MILL, $\mathcal{L}_{\mathsf{MILL}}$, then becomes

$$A ::= \mathbf{1} \mid p \mid A \otimes A \mid A \multimap A \mid \Box A \qquad \text{where } p \in Atom.$$

We add a neighborhood function [Chellas 1980] to Kripke resource models:

$$N : M \to \mathcal{P}(\mathcal{P}(M))$$

Truth condition:

$m \models \Box A$ iff $\|A\| \in N(m)$

$m \models A \multimap B$ iff $\forall n$, if $n \models A$, then $m \circ n \models B$.

$m \models A \otimes B$ iff $\exists m_1, m_2$, s.t. $m_1 \circ m_2 \prec m$, $m_1 \models A$, $m_2 \models B$.

$m \models A \,\&\, B$ iff $m \models A$ and $m \models B$.

Definition

A *modal Kripke resource model* is a structure $\mathcal{M} = (M, e, \circ, \prec, N, V)$ such that:

- $(M, e, \circ, \prec)$ is a Kripke resource frame;
- N is a neighborhood function such that:

$$\text{if } X \in N(m) \text{ and } n \prec m \text{ then } X \in N(n)$$

- V is a valuation function.

It is readily checked every $A \in \mathcal{L}_{\mathrm{MILL}}$ satisfies heredity:

Proposition

For every formula A, if $m \models A$ and $m \prec m'$, then $m' \models A$.

Sequent calculus $\vdash$

Γ and Γ' are finite multisets of formulas. (Exchange rule holds implicitly.)

$$\dfrac{}{A \vdash A}\ \text{ax} \qquad \dfrac{\Gamma, A \vdash C \qquad \Gamma' \vdash A}{\Gamma, \Gamma' \vdash C}\ \text{cut}$$

$$\dfrac{\Gamma, A, B \vdash C}{\Gamma, A \otimes B \vdash C}\ \otimes\text{L} \qquad \dfrac{\Gamma \vdash A \qquad \Gamma' \vdash B}{\Gamma, \Gamma' \vdash A \otimes B}\ \otimes\text{R} \qquad \dfrac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \& B}$$

$$\dfrac{\Gamma \vdash A \qquad \Gamma', B \vdash C}{\Gamma', \Gamma, A \multimap B \vdash C}\ \multimap\text{L} \qquad \dfrac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}\ \multimap\text{R}$$

$$\dfrac{\Gamma \vdash C}{\Gamma, \mathbf{1} \vdash C}\ \mathbf{1}\text{L} \qquad \dfrac{}{\vdash \mathbf{1}}\ \mathbf{1}\text{R}$$

$$\dfrac{A \vdash B \qquad B \vdash A}{\Box A \vdash \Box B}\ \Box(\text{re})$$

- $A \multimap A$
- $(A \multimap B) \multimap ((B \multimap C) \multimap (A \multimap C))$
- $A \multimap (B \multimap C) \multimap (B \multimap (A \multimap C))$
- $A \multimap (B \multimap A \otimes B)$
- $A \multimap (B \multimap C) \multimap A \otimes B \multimap C$
- $\mathbf{1}$
- $\mathbf{1} \multimap (A \multimap A)$

$\multimap$-rule: if $\vdash_{\mathsf{H}} A$, $\vdash_{\mathsf{H}} A \multimap B$ then $\vdash_{\mathsf{H}} B$

$\Box$(re): if $\vdash_{\mathsf{H}} A \multimap B$ and $\vdash_{\mathsf{H}} B \multimap A$ then $\vdash_{\mathsf{H}} \Box A \multimap \Box B$

Theoretical results I

Theorem

Correspondences

- *if* $\models \Gamma^* \multimap A$, *then* $\Gamma \vdash A$ *(completeness of* $\vdash$*)*
- $\Gamma \vdash A$ *iff* $\Gamma \vdash_{\mathrm{H}} A$ *(equivalence of Sequents and Hilbert)*
- *if* $\Gamma \vdash_{\mathrm{H}} A$, *then* $\models \Gamma^* \multimap A$ *(soundness of* $\vdash_{\mathrm{H}}$*)*

Where $\Gamma^\star = \varphi_1 \otimes \cdots \otimes \varphi_n$, for $\varphi_i \in \Gamma$.

Theorem

Cut elimination holds for MILL.

I.e. there exists a normal form of proofs of the sequent calculus.

E.g., proof by rediuction steps:

$$\dfrac{\dfrac{B \vdash C \qquad C \vdash B}{\Box B \vdash \Box C}\,\Box(\text{re}) \qquad \dfrac{C \vdash D \qquad D \vdash C}{\Box C \vdash \Box D}\,\Box(\text{re})}{\Box B \vdash \Box D}\,\text{cut}$$

is reduced by replacing the cut on $\Box C$ by two “lesser cuts” on C.

$$\dfrac{\dfrac{B \vdash C \qquad C \vdash D}{B \vdash D}\,\text{cut} \qquad \dfrac{D \vdash C \qquad C \vdash B}{D \vdash B}\,\text{cut}}{\Box B \vdash \Box D}\,\Box(\text{re})$$

Theoretical results III

Theorem

Proof search complexity for MILL *is in PSPACE.*

This is a membership result, due to the chose fragment of intuitionistic multiplicative additive linear logic (MALL). A hardness result holds for the full MALL.

$\Box_a A$: "agent a brings about A."

Principles:

1. If two statements are equivalent, then bringing about one is equivalent to bringing about the other.
   - if $\vdash_H A \multimap B$ and $\vdash_H B \multimap A$ then $\vdash_H \Box_a A \multimap \Box_a B$
2. If something is brought about, then this something holds.
   - $\Box_a A \multimap A$
3. It is not possible to bring about a tautology.
   - if $\vdash_H A$ then $\vdash_H \Box_a A \multimap \bot$
4. If an agent brings about two things concomitantly then the agent also brings about the conjunction of these two things.
   - $\Box_a A \otimes \Box_a B \multimap \Box_a (A \otimes B)$

Sequent rules (extends the calculus for ILL)

Linear BIAT

$$\frac{A \vdash B \qquad B \vdash A}{\Box_a A \vdash \Box_a B}\ \Box_a(\mathrm{re})$$

$$\frac{\Gamma, A \vdash B}{\Gamma, \Box_a A \vdash B}\ \mathrm{act}(a) \qquad \frac{\Gamma \vdash \Box_a A \qquad \Delta \vdash \Box_a B}{\Gamma, \Delta \vdash \Box_a(A \otimes B)}\ \Box_a\otimes$$

$$\frac{\vdash A}{\Box_a A \vdash \bot}\ \sim\mathrm{nec}$$

Theorem (Completeness)

The sequent calculus (Hilbert system) for Linear BIAT is sound and complete wrt the class of modal Kripke resource frames that satisfy:

1. ($\sim$nec) requires: if $X \in N_a(w)$ and $e \in X$, then $w \in V(\perp)$.
2. (act(a)) requires: if $X \in N_a(w)$, then $w \in X$.
3. Define $X \circ Y = \{x \circ y \mid x \in X \text{ and } y \in Y\}$ and $X^{\uparrow} = \{y \mid y \geq x \text{ and } x \in X\}$:

   ($\Box_a \otimes$) requires: if $X \in N_a(x)$ and $Y \in N_a(y)$, then $(X \circ Y)^{\uparrow} \in N_a(x \circ y)$.

Theoretical results for Linear BIAT II

Theorem (Cut elimination)

Cut elimination holds for Linear BIAT

Theorem

Proof search complexity for Linear BIAT is in PSPACE

An application: Artefacts

- Artefacts are special kind of objects that are characterized by the fact that they are designed to achieve a goal in a particular environment.
- An important aspect of artefacts is their interaction with the environment: with the *agents* that use artefacts to achieve a specific goal and with the *resources* required to achieve that goal.
- A *logical* modeling of artefacts means:
- the function of the artefact is represented by a logical formula;
- the behaviour of the artefact in the environment is captured by means of a form of reasoning.

Very simple artefact and person-artefact interaction

An electric screwdriver has two components:

- A power-pistol (p) produces some rotational force (F) when the button is pushed (P): $\Box_p(P \multimap F)$.
- The screwdriver bit (b) tightens a loose screw (S) when a rotational force (F) is applied: $\Box_b(S \otimes F \multimap T)$.

Suppose that:

- We have an electric screwdriver (p and b);
- We have a loose screw (S);
- agent a pushes the button of the pistol ($\Box_a P$).

- We can have a tighten screw (T), as the provability of the following sequent shows:

$$S, \Box_a P, \Box_p(P \multimap F), \Box_b(S \otimes F \multimap T) \vdash T$$

$$\dfrac{\dfrac{\dfrac{S \vdash S \qquad \dfrac{\dfrac{P \vdash P}{\Box_a P \vdash P}\,\Box_a(\text{refl}) \qquad F \vdash F}{\Box_a P, P \multimap F \vdash F}\,\multimap\text{L}}{S, \Box_a P, P \multimap F \vdash S \otimes F}\,\otimes\text{R} \qquad T \vdash T}{S, \Box_a P, P \multimap F, S \otimes F \multimap T \vdash T}\,\multimap\text{L}}{S, \Box_a P, \Box_p(P \multimap F), \Box_b(S \otimes F \multimap T) \vdash T}\,\ldots$$

- We cannot have two tighten screws:

$$S, \Box_a P, \Box_p(P \multimap F), \Box_b(S \otimes F \multimap T)) \nvdash T \otimes T$$

- Formulas encode the function of the artefact
- Provability shows the achievability of the goal in a given environment
- A proof encodes then the actualisation of the behaviour of the artefact in a given environment. The execution of the artefact in a given context.

Order-sensitivity I

We can drop the exchange rule, too.

$$\frac{\Gamma, A, B, \Delta \vdash C}{\Gamma, B, A, \Delta \vdash C}\ (E)$$

Extension to partially commutative Linear Logic (PCL) (De Groote, Retoreé, et al):

- non-commutative multiplicative conjunction $\odot$:
  - $A \odot B$ reads "first A then B";
- two order sensitive linear implications $\backslash$ and $/$ (like in Lambek Calculus).

Talking about sequences of actions in the logics of actual agency:

- $\Box_a(A \odot B)$ reads "a does A followed by B";
- $(\Box_a A) \odot (\Box_b B)$ reads "first a brings about A then b brings about B".

The language of modal PCL extends the language of MILL by adding the following operators:
The non-commutative tensor noted by $\odot$ and the two order sensitive implications noted $\backslash$ and $/$:

$$A ::= \mathbf{1} \mid p \mid A \otimes A \mid A \,\&\, A \mid A \multimap A \mid A \odot A \mid A \backslash A \mid A/A \mid \Box A$$

where $p \in Atom$.

PCL - Structural rules

$$\frac{\Gamma[\Delta_1,(\Delta_2,\Delta_3)] \vdash A}{\Gamma[(\Delta_1,\Delta_2),\Delta_3)] \vdash A}\ ,\mathrm{a1} \qquad \frac{\Gamma[(\Delta_1,\Delta_2),\Delta_3)] \vdash A}{\Gamma[\Delta_1,(\Delta_2,\Delta_3)] \vdash A}\ ,\mathrm{a2}$$

$$\frac{\Gamma[\Delta_1;(\Delta_2;\Delta_3)] \vdash A}{\Gamma[(\Delta_1;\Delta_2);\Delta_3)] \vdash A}\ ;\mathrm{a1} \qquad \frac{\Gamma[(\Delta_1;\Delta_2);\Delta_3)] \vdash A}{\Gamma[\Delta_1;(\Delta_2;\Delta_3)] \vdash A}\ ;\mathrm{a2}$$

$$\frac{\Gamma[\Delta_1,\Delta_2] \vdash A}{\Gamma[\Delta_2,\Delta_1] \vdash A}\ ,\mathrm{com} \qquad \frac{\Gamma[\Delta_1;\Delta_2] \vdash A}{\Gamma[\Delta_1,\Delta_2] \vdash A}\ \mathrm{ent}$$

$$\frac{\Gamma[A;B] \vdash C}{\Gamma[A \odot B] \vdash C}\odot L \qquad \frac{\Gamma \vdash A \qquad \Gamma' \vdash B}{\Gamma;\Gamma' \vdash A \odot B}\odot R$$

$$\frac{\Gamma \vdash A \qquad \Delta[B] \vdash C}{\Delta[\Gamma; A \setminus B] \vdash C}\setminus L \qquad \frac{A;\Gamma \vdash B}{\Gamma \vdash A \setminus B}\setminus R$$

$$\frac{\Gamma \vdash A \qquad \Delta[B] \vdash C}{\Delta[B/A;\Gamma] \vdash C}/\, L \qquad \frac{\Gamma; A \vdash B}{\Gamma \vdash A/B}/R$$

The axioms of MILL hold, plus: we have to rephrase the axiom $\Box_a$(refl) for mixed commutative and non-commutative context and add an axiom for combining sequences of actions:

$$\frac{\Gamma[A] \vdash B}{\Gamma[\Box_a A] \vdash B}\ \Box_a(\text{refl}) \qquad \frac{\Gamma \vdash \Box_a A \qquad \Delta \vdash \Box_a B}{\Gamma; \Delta \vdash \Box_a (A \odot B)}\ \Box_a \odot$$

- The model is now specified by $\mathcal{M} = (M, e, \circ, \bullet, \geq, N, V)$.
- Bifunctoriality is assumed also for $\bullet$: if $m \geq n$, and $m' \geq n'$, then $m \bullet m' \geq n \bullet n'$.
- The entropy principle is captured in the models by means of the following constraint: for all x, y, $x \circ y \geq x \bullet y$.
- If $\mathcal{M}$ satisfies all these conditions, we call it a *partially commutative modal Kripke resource model*.

The new truth conditions are the following:

$m \models_{\mathcal{M}} A \odot B$ iff there exist m_1 and m_2 such that $m \geq m_1 \bullet m_2$ and $m_1 \models_{\mathcal{M}} A$ and $m_2 \models_{\mathcal{M}} B$.

$m \models_{\mathcal{M}} A \setminus B$ iff for all $n \in M$, if $n \models_{\mathcal{M}} A$, then $n \bullet m \models_{\mathcal{M}} B$.

$m \models_{\mathcal{M}} B/A$ iff for all $n \in M$, if $n \models_{\mathcal{M}} A$, then $m \bullet n \models_{\mathcal{M}} B$.

Note: if $m \models A \otimes B$, then by $m_1 \circ m_2 \geq m_1 \bullet m_2$ and heredity, we have $m \models A \odot B$.

$$\dfrac{\dfrac{S \vdash S \qquad \dfrac{F \vdash F}{\Box_i F \vdash F}\,\Box_i(\text{refl})}{S; \Box_i F \vdash S \odot F}\,\odot\text{R} \qquad T \vdash T}{\dfrac{(S; \Box_i F); S \odot F \setminus T \vdash T}{S; \Box_i F; \Box_{s\bullet}(S \odot F \setminus T) \vdash T}\,\Box_{s\bullet}(\text{refl})}\,\setminus\text{L}$$

- A logic for reasoning about resource sensitive-agency: the linear propositional part takes care of resource dependencies and the modal part accounts for agency.
- We presented a number of results that show that MILL and Linear BIAT are logically well behaved systems.
- Future work:
  - Extend the intuitionistic fragment to full classical linear logic, relvant logics, bunch implication.
  - Provide a resource-sensitive account of a number of logic of agency, e.g. Coalition Logic.

A logical modelling of resource sensitive planning:

- Formulas encode the resource sensitive description of the actions and the goals.
- Provability shows the achievability of the goal in a given environment
- A proof encodes then the execution of the goal in a given environment.

An automatically generated plan[1]

```
prove> [b](S*F->T),[p](P->F),[a]P,S ==> T

          ------- Ax
          P ==> P
          ----------  act([a])  ------- Ax
          [a]P ==> P            F ==> F
          ----------------------------- L->
                  P->F, [a]P ==> F
------- Ax        ---------------------  act([p])
S ==> S           [p](P->F), [a]P ==> F
------------------------------------ R*   ------- Ax
    [p](P->F), [a]P, S ==> S*F            T ==> T
    ------------------------------------------- L->
         S*F->T, [p](P->F), [a]P, S ==> T
         ---------------------------------------- act([b])
         [b](S*F->T), [p](P->F), [a]P, S ==> T
```

[1] http://www.loa.istc.cnr.it/personal/troquard/SOFTWARES/MLLPROVER/mllprover.html

A logical modelling of agent-based economic actions (negotiation) ([PE10a, PE10b]):

- Formulas encode economic actions (buying, selling, etc).
- Provability of a suitable sequent means the possibility of concluding a deal.
- A proof of the sequent encodes the execution of the economic deal.

$$\underbrace{p_1, \ldots, p_m}_{goods}, \underbrace{\bigotimes_{p \in \mathcal{A}} [\&_{j \in \mathcal{N}} (p \multimap \Box_j p)]^{\mathcal{M}(p)}}_{who\ gets\ what}, \underbrace{\text{PREF}_1, \ldots, \text{PREF}_n}_{Agents'\ preferences} \vdash \underbrace{u^k}_{revenue}$$

where PREF_j is a formula expressing buying or selling items involving j, e.g.

$$\Box_j (p \otimes q \otimes r) \to u^j$$

- We discussed the foundation of the modelling of actions in the tradition of the logic of agency.
- We focussed in particular on STIT and on BIAT.
- We proposed a reformulation of BIAT in the realm of resource sensitive logics.
- We investigated the theoretical properties of those logics.
- We discussed a few modelling examples of the application of a resource sensitive logic of agency.

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