Strategizing in environments with common-pool resources

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Commons

Common-pool resources are resources like water, air, pastures, or fish stocks.

- They are non-excludable: they are out there for the taking.
- They are rivalrous: one agent's consumption can limit or prevent another agent to consume it.




Garrett Hardin: Freedom in a commons brings ruin to all. (Hardin 1968)[1]

Elinor Ostrom: Evidence of wide-ranging instances of sustainable commons, and identification of design principles of successful common-pool resource management (Ostrom 1990)[2].

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[1] Garrett Hardin. "The Tragedy of the Commons". In: *Science* 162 (1968), 1243—1248.

[2] Elinor Ostrom. *Governing the Commons: The evolution of institutions for collective action*. Cambridge University Press, 1990.

A commons: the memory heap

Thread agents

thread1(actions1, objective1) { . . . malloc . . . free . . . }
thread2(actions2, objective2) { . . . malloc . . . free . . . }
thread3(actions3, objective3) { . . . malloc . . . free . . . }

Robots in the commons

Past: robots in "private goods economies"	**Future robots in the commons**
robots in labs, robots in industrial environments	robots intruding the physical world
They can rely on direct wires to an electricity source, human operator providing raw materials.	They autonomously manage the multiple resources they need. They must carefully consume them, and in presence of competitors, they must also be careful in how they produce them.

Algorithmic management of the commons

General aim: contribute to the pursued efforts of engineering solutions for the sustainable management of commons.

Resource contribution games (*second part of this talk*):

- determine the rational behaviours of agents with resource endowments and resource objectives.

Two problem of synthesis (Fisman et al. 2010)[3] (*first part of this talk*):

- Rational synthesis: recommend a behaviour to all the agents in a commons, desirable from the point of view of the system, that the agents have individually no incentive to reject.
- Non-cooperative rational synthesis: find a strategy for a controller, such that for all rational answers by the agents in a commons, the outcome is desirable from the point of view of the system.

[3] Dana Fisman, Orna Kupferman, and Yoad Lustig. "Rational Synthesis". In: *TACAS*. 2010.

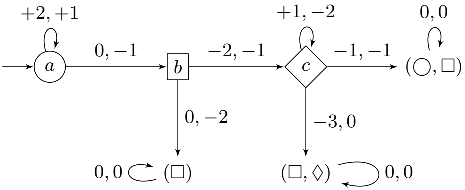
Setting

R. Condurache, C. Dima, M. Jitaru, Y. Oualhadj, N. Troquard. Careful Autonomous Agents in Environments With Multiple Common Resources. *AREA@IJCAI'22.*

The agents interact in a turn-based fashion in an arena: a graph where in each state one agent decides the next edge to follow.

Each agent has a qualitative temporal objective.

To each edge is associated a tuple of integers corresponding to the resource cost/reward when taking that action.



The agents may be concerned with a quantitative objective as well.

Agent interactions = strategies and strategy profiles

A strategy for a player specifies an edge to take for every game history ending in a state where this player decides.

- Particular case: memoryless.

A strategy profile specifies one strategy for every player.

Controller/strategy synthesis

- Setting: A game with two entities in opposition: a system controller and a system environment. A specification for the system is chosen.
- Problem: Automatically design a controller/strategy for a system that will enforce the specification (Church, 1957)[4], (Büchi, Landweber, 1969)[5], (Pnueli and Rosner, 1989)[6].

Strategy synthesis is pessimistic: The environment is a perfect antagonist to the controller. (Two-player zero-sum game.) What if the agents making up the environment had their own objectives?

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[4] Alonzo Church. "Applications of recursive arithmetic to the problem of circuit synthesis". In: *Summaries of the Summer Institute of Symbolic Logic* 1 (1957).

[5] J.R. Büchi and L.H. Landweber. "Solving sequential conditions by finite-state strategies". In: *Trans. Trans. Amer. Math. Soc.* (1969).

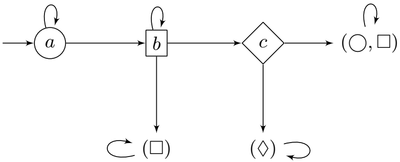
[6] Amir Pnueli and Roni Rosner. "On the Synthesis of a Reactive Module". In: *POPL*. 1989.

Rationality

Each agent has a qualitative temporal objective:

- Parity automata: canonical definitions for languages of infinite words
- Particular cases: Büchi automata
- Linear-time Temporal Logic (LTL) formula: $\mathsf{X}\phi$, $\phi \mathsf{U} \psi$, $\ldots$
- Particular cases: reachability, safety, ...

A strategy profile is a Nash equilibrium if no player can unilateraly and profitably change his strategy.



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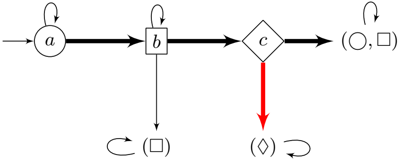

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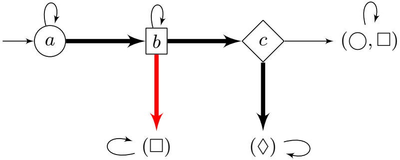


Outline

Rational synthesis in the commons

arena with G → **Rational synthesis**

objectives $\phi_1, \ldots, \phi_n$ → **Rational synthesis**

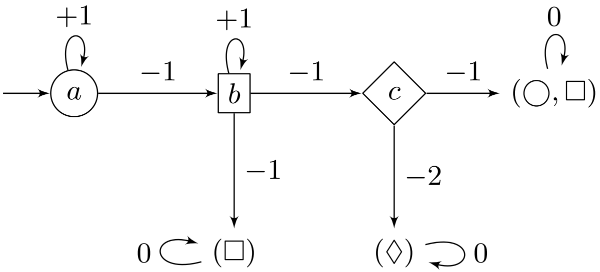
Rational synthesis → A profile $S = (S_1, S_2, \ldots, S_n)$, s.t.

- S is a NE,
- $G_{|S} \models \phi_1$, and
- $G_{|S} \in \text{OkResources}(G)$.

- Find a strategy for agent 1 such that in the sub-game that it defines, there exists a Nash Equilibrium whose outcome satisfies agent 1's objective, and never depletes any system resource.
- Agent 1 holds the special role of controller, and is always careful.
- Two flavors: careless agents, careful agents.
  - Careless agents bother only about their temporal objective.
    - Impact on NE : A careless agent may deviate to satisfy her temporal objective even if this causes some resource to drop below zero.
  - Careful agents also pay attention to not deplete the system's resources.
    - Impact on NE : A careful agent may deviate when she finds a way to satisfy her temporal objective, and no resources drop below zero.

Careless and careful solutions, single resource case

R. Condurache, C. Dima, Y. Oualhadj, N. Troquard. Rational Synthesis in the Commons with Careless and Careful Agents. *AAMAS'21.*



Careless and careful solutions, single resource case

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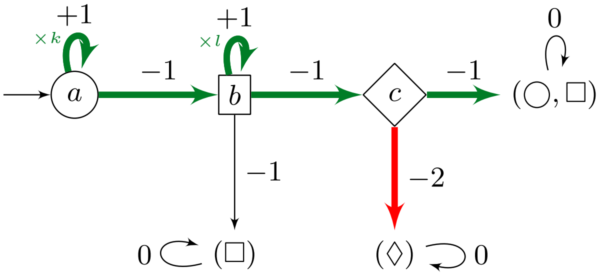
Careless :



There is NO careless solution to the rational synthesis problem with agent $1 = \bigcirc$.

Careless and careful solutions, single resource case

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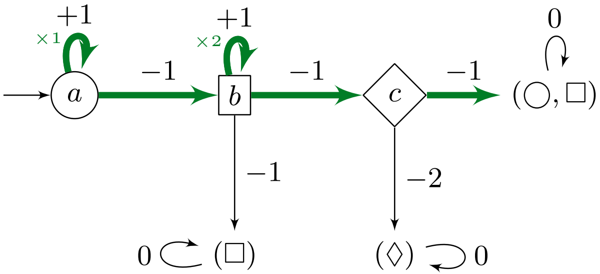
Careful :



There exists a careful solution to the rational synthesis problem

Complexity of the rational synthesis problem, single resource case

R. Condurache, C. Dima, Y. Oualhadj, N. Troquard. Rational Synthesis in the Commons with Careless and Careful Agents. *AAMAS'21.*

	no resources [7,8]	careless	careful
Parity	NP-c	NP-c	PSPACE-c
Büchi	PTIME-c	PTIME-c	PSPACE-c

Proof idea (case parity – careful):

- Energy-parity games.[9] $\mathsf{Win}_i = \{s \in S \mid \mathsf{MinCredit}(s,i) \neq \omega\}$
- Constraint LTL.[10] $\Phi_{\mathsf{prty}_i} = \bigwedge_{\substack{s \in S \\ \mathsf{prty}_i(s) \text{ is odd}}} \left(\Box\Diamond s \rightarrow \bigvee_{\substack{s' \in S \\ \mathsf{prty}_i(s') < \mathsf{prty}_i(s) \\ \mathsf{prty}_i(s') \text{ is even}}} \Box\Diamond s' \right)$

$$\langle \sigma_1, \ldots, \sigma_n \rangle \models \Phi_{\mathsf{prty}1} \wedge \Box(x \geq 0) \wedge \bigwedge_i \left(\neg\Phi_{\mathsf{prty}i} \rightarrow \Box\neg\left(\bigvee_{s \in \mathsf{Win}_i} s \wedge (x \geq \mathsf{MinCredit}(s,i)) \right) \right)$$

- One-counter bounded automata.[11]

[7] Michael Ummels. "The Complexity of Nash Equilibria in Infinite Multiplayer Games". In: *Foundations of Software Science and Computational Structures.* 2008.

[8] Rodica Condurache et al. "The Complexity of Rational Synthesis". In: *ICALP.* 2016.

[9] Krishnendu Chatterjee and Laurent Doyen. "Energy parity games". In: *Theor. Comput. Sci.* (2012).

[10] Stéphane Demri and Régis Gascon. "The Effects of Bounding Syntactic Resources on Presburger LTL". In: *J. Log. Comput.* 19.6 (2009), pp. 1541–1575.

[11] John Fearnley and Marcin Jurdzinski. "Reachability in two-clock timed automata is PSPACE-complete". In: *Inf. Comput.* 243 (2015), pp. 26–36.

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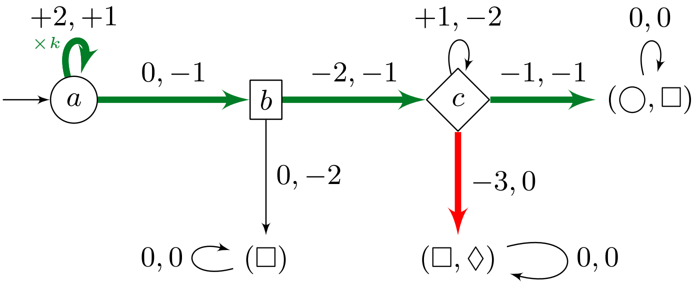
[7] Ummels, "The Complexity of Nash Equilibria in Infinite Multiplayer Games".
[8] Condurache et al., "The Complexity of Rational Synthesis".
[9] Chatterjee and Doyen, "Energy parity games".
[10] Demri and Gascon, "The Effects of Bounding Syntactic Resources on Presburger LTL".
[11] Fearnley and Jurdzinski, "Reachability in two-clock timed automata is PSPACE-complete".

Multi-resource example

R. Condurache, C. Dima, M. Jitaru, Y. Oualhadj, N. Troquard. Careful Autonomous Agents in Environments With Multiple Common Resources. *AREA@IJCAI'22*.



There is NO solution (careless or careful) to the rational synthesis problem.

Undecidability

Theorem

The problem of careful rational synthesis is undecidable, even with two players and two resources, and reachability objectives.

Proof idea: Encode the reachability problem in multi-counter automata.

- Agent 1 tries to build a computation reaching some final state with all counters 0.
- Agent 2 checks that agent 1 does not cheat:
  - Each computation step satisfies the upper and lower guard of the transition chosen by agent 1.
  - The final configuration has all counters equal to 0.

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What about bounded storage of resources?

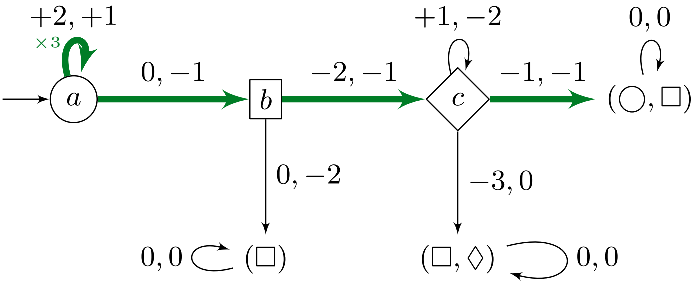
In many real-case scenarios, resources are bounded, e.g.:

- a shared tank of water can only contain a predetermined amount of water;
- a shared microgrid powerpack can only contain a predetermined amount of energy.
- . . .

Arenas with bounded resources:

- An upper-bound B_i is fixed for every resource.
- Whenever the i-th resource reaches B_i, an increment leaves it unchanged.

Multi-resource example, upper bound $B = (3,3)$



Solution to the careful rational synthesis problem, with bounds $(3,3)$.

$$(a,2,1) \to (a,3,2) \to (a,3,3) \to (b,3,2) \to (c,1,1) \to ((\bigcirc,\square),0,0)$$

Retrieving decidability by bounding the resources

Theorem

The careful rational synthesis with LTL objectives is 2EXPTIME-complete when the resources are bounded.

Proof idea: Unfold the game arena, and use cooperative rational synthesis without resources .

- Unfolding of the configuration space of the game arena into a finite state space.
  - State space $= |S| \times \prod_{1 \leq i \leq d}(B_i + 1)$, where d is the number of resources.
- Constants encoded in binary $\longrightarrow$ finite arena of exponential size w.r.t. the size of the original arena.
- Reduce the careful synthesis problem for this arena to a (non-quantitative) cooperative synthesis problem for the unfolded arena.
- Complexity trick: the cooperative synthesis problem for LTL objectives is in 2EXPTIME in the size of the LTL formulas but only in PTIME in the size of the arena. (Fisman et al. 2010)[12]
- Lower bound is also 2EXPTIME since the synthesis problem for LTL objectives is 2EXPTIME-complete.

[12] Fisman, Kupferman, and Lustig, "Rational Synthesis".

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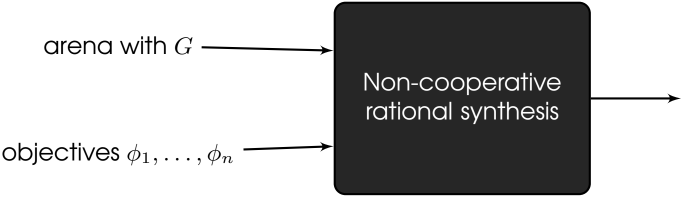
[12] Fisman, Kupferman, and Lustig, "Rational Synthesis".

Outline

Non-cooperative rational synthesis in the commons: one resource, careful controller, careless agents



A strategy S_1, s.t. for all profiles $(S_1, S_2, \ldots, S_n)$

- S is a NE,
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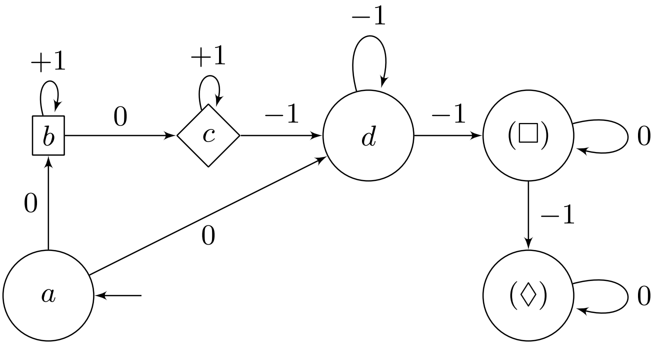
- Find a strategy for agent 1 such that in the sub-game that it defines, all Nash Equilibria's outcomes satisfy agent 1's objective, and never depletes the system resource.
- Agent 1 holds the special role of controller, and is always careful.
- The other agents are careless

Non-cooperative rational synthesis

R. Condurache, C. Dima, Y. Oualhadj, N. Troquard. Synthesis of Resource-Aware Controllers Against Rational Agents. *AAMAS'23*.



- Player 1 = $\bigcirc$, cares only about the resource. Player 2 = $\Box$ and player 3 = $\Diamond$ have reachability objectives.
- A strategy σ_1 which is a solution to the problem of non-cooperative rational synthesis is:
  - in a go to b;
  - in d go to the state labeled $\Box$ if the resource is at least 1, and loop otherwise;
  - in the state labeled $\Box$ go to the state labeled $\Diamond$ if the resource is at least 1, and loop otherwise;
  - in the state labeled $\Diamond$, loop.

Complexity of non-cooperative rational synthesis

Theorem

The non-cooperative rational synthesis with LTL objectives is 2EXPTIME-complete.

Proof idea: The hardness easily follows from classical LTL synthesis problem.
For membership:

- We study EPP games: 2-player zero-sum games with objective $(\mathsf{Energy} \cup \mathsf{Parity}_1) \cap \mathsf{Parity}_2$
- We can show that the problem of deciding the existence of a winning strategy in an EPP game is in $\mathsf{NP} \cap \mathsf{co}\text{-}\mathsf{NP}$, by a reduction to (Chatterjee and Doyen, 2012)[13]; runs in time polynomial in the size of the input arena and exponential in the size of the priority functions
- We can translate LTL objectives into a Parity automaton of size exponential, but only polynomial parities
- We can translate the problem of non-cooperative rational synthesis with Parity objectives into a problem of deciding the existence of a winning strategy in a EPP game (analogous to Suspect Games from (Bouyer et al. 2015)[14])

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[13] Chatterjee and Doyen, "Energy parity games".

[14] Patricia Bouyer et al. "Pure Nash Equilibria in Concurrent Deterministic Games". In: *Log. Methods Comput. Sci.* 11.2 (2015).

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[13] Chatterjee and Doyen, "Energy parity games".
[14] Bouyer et al., "Pure Nash Equilibria in Concurrent Deterministic Games".

Perspectives

- Cooperative case. Extend to LTL objectives.
- Non-cooperative case.
  - Extend to other ω-regular objectives.
- Concurrent case.
  - Extend *R. Condurache, Y. Oualhadj, N. Troquard. The complexity of rational synthesis for concurrent games. CONCUR'18* with resources.
- Parameterized synthesis: leave some quantities unspecified and synthesize bounds for those quantities that ensure the existence of a solution.

Outline

Example

You participate in a bring-your-own-food cooking party.

You bring a bottle  of white wine. You cannot take it out and open it without sharing it, and if other guests also want a taste of it, or to cook with it, you might not have enough to your satisfaction. Once at the party you can strategically decide whether to contribute the bottle to the party or keep it away in your bag.

Your bottle of wine contains four glasses ( =  ·  ·  · ), and your objective is to enjoy two glasses.

Bruno and Carmen both want a portion of risotto, each portion needing a portion of rice  , one onion  , and a glass of white  wine. Bruno brings three portions of rice, and Carmen brings two onions.

What are the possible equilibria?

The players are endowed with a multiset (of bundles) of resources, that cannot be used directly, but can be added into a common pool of resources.

A possible action of a player consists in providing a submultiset of his endowment to the common pool.

Each player then has a resource objective which can be attained by using the resources contributed by all the players.

In a way, the game dynamics obliges the players to redistribute their endowed resources in a competitive manner.

Example

- one glass of wine w
- one portion of rice r
- one onion o

The bottle of wine  is represented as the bundle w • w • w • w of four glasses of wine.
A portion of risotto is represented as the bundle r • o • w of one portion of rice, one onion, and one glass of wine.

The resource contribution game $\tilde{G}$ is:

- players $\{y, b, c\}$
- objectives
  - $\gamma_y = \mathsf{w} \bullet \mathsf{w}$
  - $\gamma_b = \mathsf{r} \bullet \mathsf{o} \bullet \mathsf{w}$
  - $\gamma_c = \mathsf{r} \bullet \mathsf{o} \bullet \mathsf{w}$
- endowments
  - $\epsilon_y = \{\mathsf{w} \bullet \mathsf{w} \bullet \mathsf{w} \bullet \mathsf{w}\}$
  - $\epsilon_b = \{\mathsf{r}, \mathsf{r}, \mathsf{r}\}$
  - $\epsilon_c = \{\mathsf{o}, \mathsf{o}\}$

Example

The resource contribution game $\tilde{G}$ is:

Player i	endowment ϵ_i	objective γ_i
you	{  .  .  .  }	 . 
Bruno	{  ,  ,  }	 .  . 
Carmen	{  ,  }	 .  . 

Potential satisfaction and resource contention

A player is potentially satisfied in an action profile if all the contributed resources can be transformed into his objective.

Example

Everyone is potentially satsified in the profile where you contribute $\{\mathrm{w} \bullet \mathrm{w} \bullet \mathrm{w} \bullet \mathrm{w}\}$ Bruno contributes $\{\mathrm{r}\}$, and Carmen contributes $\{\mathrm{o}\}$.

Player i	i's contribution	objective γ_i
you	{  }	
Bruno	{  }	
Carmen	{  }	

An action profile providing enough resources to potentially satisfy a set of players but not enough to satisfy them all at once is a contentious profile.

Example

The previous profile is contentious. Bruno and Carmen are potentially satsified in it, but one cannot make two risotti.

Attitudes about resource contention

Contention-tolerant players will not worry about contentious profiles, and find them good, as long as they can be potentially satisfied.

Contention-averse players will not consider a contentious profile good, even if they can be potentially satisfied.

A resource contention in a profile may not concern a player directly. This is the case when the contention is about a resource that is independent of the player's objective.

Private contention-averse players that are potentially satisfied in such a profile will consider it good. They will not consider good a profile in which the contention is about a resource that plays a part in their objective.

Example

The previous profile is not good for Bruno and Carmen. But it is good for you if you are private contention-averse. There's enough wine for you and two risotti.

Attitudes about surplus and parsimony

The players are surplus-tolerant, and parsimonious.

A surplus-tolerant player does not mind to have more resources available than necessary.

A parsimonious player will prefer a profile in which they contribute strictly less from another profile, if they find the profile otherwise equally good.

Nash equilibria

The players want to be potentially satisfied in a good profile, and contribute as little as possible.

A Nash Equilibrium is an action profile where no player can do better by unilaterally changing their contribution.

Example

In the case of contention-tolerance

- $\mathsf{NE}^{\mathsf{ct}}(\tilde{G}) = \{(\mathsf{w} \cdot \mathsf{w} \cdot \mathsf{w} \cdot \mathsf{w}, \emptyset, \emptyset), (\mathsf{w} \cdot \mathsf{w} \cdot \mathsf{w} \cdot \mathsf{w}, \{\mathsf{r}\}, \{\mathsf{o}\})\}$.

In both cases of contention-aversity

- $\mathsf{NE}^{\mathsf{ca}}(\tilde{G}) = \mathsf{NE}^{\mathsf{pca}}(\tilde{G}) = \{(\mathsf{w} \cdot \mathsf{w} \cdot \mathsf{w} \cdot \mathsf{w}, \emptyset, \emptyset), (\mathsf{w} \cdot \mathsf{w} \cdot \mathsf{w} \cdot \mathsf{w}, \{\mathsf{r}, \mathsf{r}\}, \{\mathsf{o}, \mathsf{o}\})\}$.

RCGs and RCGBARs

We consider two cases of Resource Contribution Games:

- RCGs: General case.
- RCGBARs: Case where endowments are bags of atomic resources.

Example

{  •  •  •  } cannot be an endowment of an RCGBAR.

{  ,  ,  ,  } can be an endowment of an RCGBAR.

Results

N. Troquard. Existence and verification of Nash equilibria in non-cooperative contribution games with resource contention. *Ann. Math. Artif. Intell., 2024.*

Complexity of deciding whether $P \in \mathsf{NE}^{?}(G)$.

?	RCG	RCGBAR
ct	PTIME ((Tr. 16, ijcai)[15])	PTIME
ca	co-NP-c	PTIME
pca	co-NP-c	PTIME

$\mathsf{NE}^{?}(G)$ always non empty?

?	RCG	RCGBAR
ct	no	no
ca	no	yes!
pca	no	no

[15] Nicolas Troquard. "Nash Equilibria and Their Elimination in Resource Games". In: *IJCAI 2016*.

RCGBAR, finding an element in $\mathsf{NE}^{\mathsf{ca}}(G)$

Theorem

With contention-adverse players, a Nash Equilibrium always exists in a RCGBAR, and finding one can be done in polynomial time.

Proof idea:

- There is a polynomial algorithm to find *minimal* profitable deviations.
- Starting from $(\emptyset, \emptyset, \ldots, \emptyset)$ we can iteratively make minimal profitable deviations.
- $(P_1^1, \emptyset, \ldots, \emptyset) \rightarrow (P_1^1, P_2^2, \ldots, \emptyset) \ldots \rightarrow (P_1^1, P_2^2, \ldots, P_n^n) \rightarrow (P_1^{n+1}, P_2^2, \ldots, P_n^n) \ldots$
- The order of the agents does not matter, but can yield different outcomes.
- This is monotonic (only for ca). It stops in polynomial time with a Nash equilibrium.

But: The Nash equilibrium found is possibly very inefficient, like, $(\emptyset, \emptyset, \ldots, \emptyset)$.

RCGBAR, finding an element in $\mathrm{NE}^{\mathrm{ca}}(G)$

Theorem

With contention-adverse players, a Nash Equilibrium always exists in a RCGBAR, and finding one can be done in polynomial time.

Proof idea:

- There is a polynomial algorithm to find *minimal* profitable deviations.
- Starting from $(\emptyset, \emptyset, \ldots, \emptyset)$ we can iteratively make minimal profitable deviations.
- $(P_1^1, \emptyset, \ldots, \emptyset) \rightarrow (P_1^1, P_2^2, \ldots, \emptyset) \ldots \rightarrow (P_1^1, P_2^2, \ldots, P_n^n) \rightarrow (P_1^{n+1}, P_2^2, \ldots, P_n^n) \ldots$
- The order of the agents does not matter, but can yield different outcomes.
- This is monotonic (only for ca). It stops in polynomial time with a Nash equilibrium.

But: The Nash equilibrium found is possibly very inefficient, like, $(\emptyset, \emptyset, \ldots, \emptyset)$.

RCGBAR, finding an element in $\mathsf{NE}^{\mathsf{ca}}(G)$

Example

- one portion of wine w
- one portion of rice r
- one onion o

Instead of one bottle  , you have 4 portions  $\times 4$ of wine.

The resource contribution game $\tilde{G}'$ is:

- players $\{y, b, c\}$
- objectives
  - $\gamma_y = \mathsf{w} \bullet \mathsf{w}$
  - $\gamma_b = \mathsf{w} \bullet \mathsf{r} \bullet \mathsf{o}$
  - $\gamma_c = \mathsf{w} \bullet \mathsf{r} \bullet \mathsf{o}$
- endowments
  - $\epsilon_y = \{\mathsf{w}, \mathsf{w}, \mathsf{w}, \mathsf{w}\}$
  - $\epsilon_b = \{\mathsf{r}, \mathsf{r}, \mathsf{r}\}$
  - $\epsilon_c = \{\mathsf{o}, \mathsf{o}\}$

The algorithm yields $(\emptyset, \emptyset, \emptyset) \to (\{\mathsf{w}, \mathsf{w}\}, \emptyset, \emptyset) \in \mathsf{NE}^{\mathsf{ca}}(\tilde{G}')$.
But $(\{\mathsf{w}, \mathsf{w}, \mathsf{w}, \mathsf{w}\}, \{\mathsf{r}, \mathsf{r}\}, \{\mathsf{o}, \mathsf{o}\}) \in \mathsf{NE}^{\mathsf{ca}}(\tilde{G}')$ is much better.

Perspectives

- Finding an element in $\mathsf{NE}^{\mathsf{ca}}(G)$ for RCGBARs.
  - Does the best-response dynamics work from profiles other than $(\emptyset, \emptyset, \ldots, \emptyset)$?
- How hard is it to find an efficient Nash equilibrium?
- Study preferences with partial potential satisfaction.
- Extend the notion of contention to Linear Logic resources (Tr 20, JLC)[16]
  - $\mathsf{menu} = \mathsf{dish} \otimes \mathsf{side} \otimes \mathsf{dessert}$
  - $\mathsf{dish} = \mathsf{fish} \,\&\, \mathsf{meat}$
  - $\mathsf{side} = \mathsf{aubergine} \oplus (\mathsf{parsnip} \otimes \mathsf{leek}) \oplus \mathsf{asparagus}$
  - $\mathsf{dessert} = (\mathsf{strudel} \,\&\, \mathsf{sachertart}) \otimes ((\$1 \multimap \mathsf{icecream}) \,\&\, \mathbf{1})$

[16] Nicolas Troquard. "Individual resource games and resource redistributions". In: *J. Log. Comput.* 30.5 (2020), pp. 1023–1062.

Strategizing in environments with common-pool resources

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ω-regular objectives

Each agent has a temporal objective over the set of states, modelled as an ω-regular langage.

An ω-regular language is either:

- A^ω, where A is a regular language (non-empty, not containing the empty string). E.g., $((a \cup b)^* c))^\omega$.
- AB, where A is a regular language and B is an ω-regular language. E.g., $(c^* ab(ab)^*)((a \cup b)^* c))^\omega$.
- $A \cup B$, when A and B are ω-regular languages. E.g., $((a \cup b)^* c))^\omega \cup (c^* ab(ab)^*)((a \cup b)^* c))^\omega$.

Büchi, coBüchi, LTL, reachability, safety, . . . are particular cases.

Deterministic parity automata and objectives

Deterministic parity automata are canonical for representing all ω-regular languages.

Let:

- $\mathcal{T} = (\mathsf{S}, \mathsf{E})$ be a transition system,
- C be a finite subset of $\mathbb{N}$ (a set of colors),
- $\mathsf{prty} \colon \mathsf{S} \to C$ (a priority/coloring function).

Let:

- π be an infinite execution of the transition system $\mathcal{T}$; an infinite word of states.
- $\mathsf{Inf}(\pi)$ is the set of states occurring infinitely often along π.

$\mathcal{T}, C, \mathsf{prty}$ accepts π when

$$\min\{\mathsf{prty}(s) \mid s \in \mathsf{Inf}(\pi)\} \text{ is even}$$

E.g., π such that $\mathsf{map}(\mathsf{prty}, \pi) = 528726518166122712123121474714747474747474747\ldots$

Deterministic Büchi automata and objectives

Special case of deterministic parity automata where $C = \{0, 1\}$.

An execution π is recognized if 0-states are visited infinitely often.

LTL objectives

Syntax:

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \mathsf{X}\phi \mid \phi \mathsf{U} \phi$$

$\Diamond\phi = \top \mathsf{U} \phi$, $\Box\phi = \neg\Box\neg\phi$.

Examples:

- Ex. (Mutual exclusion): $\Box(\neg p_1 \vee \neg p_2)$
- Ex. (A response follows every request): $\Box(req \to \Diamond res)$
- Ex. (Infinitely often) $\Box\Diamond p$
- Ex. (Eventually always) $\Diamond\Box p$