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From Coalition Logic to STIT [1]

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Abstract

STIT is a logic of agency that has been proposed in the nineties in the domain of philosophy of action. It is the logic of constructions of the form "agent a sees to it that φ". We believe that STIT theory may contribute to the logical analysis of multiagent systems. To support this claim, in this paper we show that there is a close relationship with more recent logics for multiagent systems. We focus on Pauly's Coalition Logic and the logic of the *cstit* operator, as described by Horty. After a brief presentation of Coalition Logic and a discrete-time version (including a *next* operator) of the STIT framework, we introduce a translation from Coalition Logic to the discrete STIT logic, and prove that it is correct.

Keywords: multiagent systems, agency, Coalition Logic, STIT theory, modal logic

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1 Introduction

STIT is a logic of agency that has been proposed in the nineties in the domain of philosophy of action [2]. It is the logic of constructions of the form "agent a sees to it that φ".

Several versions of this modality have been studied in the philosophical literature. Here we use the simplest one, viz. the so-called Chellas' STIT operator (*cstit*) [13]. This operator has been generalized to groups of agents in [14]. Other versions such as the more complex deliberative STIT operator can be defined from Chellas'.

The semantics of the STIT operator is based on branching time temporal structures. In this it differs from the "bringing it about" operator whose semantics is defined in terms of neighborhood models that do not refer to time [18,5,16]. As a consequence it is more appropriate to study the interaction of agency and time in a STIT setting than in a "bringing it about" setting.

Up to now, the STIT operator has been used mainly in the logical analysis of agency and its relation with deontic concepts [14,13]. However, we believe that STIT theory may contribute to the logical analysis of multiagent systems in general. To support this claim, we show in this paper that there is a close relationship with more recent logics for multiagent systems.

We focus on Pauly's Coalition Logic (CL) [17]. CL has been introduced to reason about what single agents and groups of agents are able to achieve. $[A]\varphi$ reads "group A can enforce an outcome state satisfying φ". As shown by Goranko in [9], CL is a fragment of Alternating-time Temporal Logic (ATL) that has been proposed by Alur et al. [1]. In this paper we propose a translation from CL to a discrete version of STIT that includes a *next* operator.

In [19], a close examination of the differences and similarities of the models of STIT theory and ATL is undertaken. It is shown that, under the addition of some specific conditions, the models of the two systems can be seen to obey similar properties. However, these properties are not necessarily expressible in the logics of STIT or ATL. So, although from a philosophical point of view, it may be interesting to look at properties of models as such, here we are interested in those properties that are expressible in the logics. While [19] compares the models underlying the logics of ATL (and thus CL) and STIT, we directly compare the logics of both systems and give a translation.

In Section 2 we offer a brief presentation of Coalition Logic. Section 3 deals with an adapted discrete-time STIT framework. Section 4 presents the main result of this note: we describe a translation from CL to STIT, and prove that it is correct. We discuss it in Section 5. Section 6 concludes with some perspectives of investigations.

2 Coalition Logic

In what follows, $\mathcal{A}tm$ represents a set of atomic propositions, and $\mathcal{A}gt$ is a nonempty set of agents.

A *game model* is a tuple $\mathcal{M} = \langle W, \{\Sigma_{a,w} | a \in \mathcal{A}gt, w \in W\}, o, v\rangle$, where:

- W is a nonempty set of possible worlds (alias moments or states).
- $\Sigma_{a,w}$ is a nonempty set of choices (alias actions) for each agent $a \in \mathcal{A}gt$ and moment $w \in W$. From some (abstract) set of actions, a *particular* choice $\sigma_{A,w}$ of a *group* of agents $A \subseteq \mathcal{A}gt$ in a world w is defined as $\sigma_{A,w} \in \prod_{a\in A} \Sigma_{a,w}$.
- o is a function $o : \prod_{a\in\mathcal{A}gt} \Sigma_{a,w} \mapsto W$ yielding a unique outcome state for every combination of choices by agents in $\mathcal{A}gt$.
- v is a valuation function $v : \mathcal{A}tm \mapsto 2^W$.

If every agent in $\mathcal{A}gt$ opts for an action, the next state of the world is completely determined. Following Pauly, as the occasion arises we slightly generalize the type of the function o, such that it may take two arguments; $o(\sigma_{A,w}, \sigma_{(\mathcal{A}gt\setminus A),w})$ then yields the unique outcome state where the agents in $A \subseteq \mathcal{A}gt$ choose $\sigma_{A,w}$ and the agents in the complementary set $\mathcal{A}gt \setminus A$ choose $\sigma_{(\mathcal{A}gt\setminus A),w}$. Now we can generalize the function o such that it maps moments and arbitrary choices of groups A into a set of possible outcome states, by defining: $o(\sigma_{A,w}) = \{o(\sigma_{A,w}, \sigma_{(\mathcal{A}gt\setminus A),w}) \mid \sigma_{(\mathcal{A}gt\setminus A),w} \in \prod_{a\in(\mathcal{A}gt\setminus A)} \Sigma_{a,w}\}$.

Figure 1 shows an example. At moment w_0, agent a has the choice between repairing a broken lamp (ρ_a) or remaining passive (λ_a). Agent b has the vacuous choice of remaining passive : (λ_b). If a chooses not to repair, the system reaches w_1. If a chooses to repair, the system reaches w_2. In both w_1 and w_2 both agents can choose to toggle a light switch or not. So, agent a can choose to toggle (τ_a) or not (λ_a), and agent b can choose to toggle (τ_b) or not (λ_b).

Relation with Pauly's original game structures

Pauly defines the semantics of CL using models $\mathcal{M} = (W, E, V)$, where W is a nonempty set of states, E is a playable effectivity function $W \mapsto (2^{\mathcal{A}gt} \mapsto 2^{2^W})$ yielding for every state a function mapping sets of agents A to actions, understood as the set of states A's simultaneous actions result in. Playable effectivity functions are defined to obey some specific conditions, making CL frames equivalent to game frames (as Pauly proves).

The above definition of game structures differs from Pauly's in two points. First of all, we do not have the agent names as a separate set in the models. Also in the STIT models we define in Section 3, contrary to usual practice in

STIT semantics, we do not include the set of agents in the models. This is not necessary, since the agent domains of the functions $\Sigma_{a,w}$ and o are simply assumed to consist of all agents relevant for the interpretation of formulas (like the domain of the valuation function v is assumed to consist of all proposition symbols relevant for the interpretation of formulas). The other difference is that Pauly uses action sets Σ_a, while we make these sets not only relative to agents, but also to worlds (i.e. $\Sigma_{a,w}$). This difference is only cosmetical. Pauly uses one set of choices (choice names) per agent (Σ_a) that is *reused* in every world. We do not reuse choices (choice names), but use a separate set $\Sigma_{a,w}$ for every agent/world pair instead. The underlying philosophical question is whether or not two choices are always different when performed in different worlds. It is quite easy to see that the two ways of referring to choices do not have any influence on the logic. In CL (and in ATL), the actions (choices) are not made explicit in the object language. Therefore, the logic does not depend on the way we name or refer to actions (choices) in the models. The only difference then seems that in Pauly's setting, the *number* of choices in every state of a model is the same, while in our setting this is not necessarily the case. But also this is not essential, since, without affecting satisfiability, in any of our models we can always use dummy choices (e.g. duplicates of existing choices) to make the number of choices equal for each world.

Truth conditions

A formula is evaluated with respect to a model and a moment.

$$\mathcal{M}, w \models p \iff w \in v(p), p \in \mathcal{A}tm$$

$$\mathcal{M}, w \models \neg\varphi \iff \mathcal{M}, w \not\models \varphi$$

$$\mathcal{M}, w \models \varphi \vee \psi \iff \mathcal{M}, w \models \varphi \text{ or } \mathcal{M}, w \models \psi$$

The intuitive interpretation of a formula $[A]\varphi$ is that the group of agents A can enforce, in one move, an outcome moment satisfying φ. We define the semantics of the modality as follows:

$$\mathcal{M}, w \models [A]\varphi \iff \exists \sigma_{A,w} \in \prod_{a\in A} \Sigma_{a,w}, \forall w' \in o(\sigma_{A,w}), \mathcal{M}, w' \models \varphi.$$

As usual, $\models_{CL} \varphi$ denotes that $\mathcal{M}, w \models \varphi$ for every CL model $\mathcal{M}$ and world w in $\mathcal{M}$.

The following complete axiomatization of CL is given in [17]:

($\bot$) $\neg[A]\bot$

($\top$) $[A]\top$

(N) $\neg[\emptyset]\neg\varphi \rightarrow [\mathcal{A}gt]\varphi$

(M) $[A](\varphi \wedge \psi) \rightarrow [A]\varphi$

(S) $[A_1]\varphi \wedge [A_2]\psi \to [A_1 \cup A_2](\varphi \wedge \psi)$ if $A_1 \cap A_2 = \emptyset$

(RE) from $\varphi \equiv \psi$ infer $[A]\varphi \equiv [A]\psi$

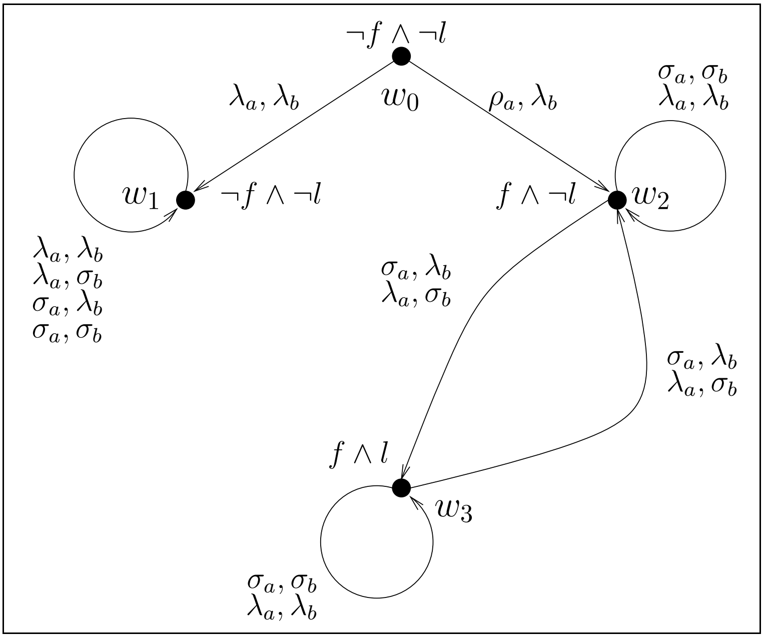

Fig. 1. Example of CL model.

On Figure 1, the proposition f stands for "the light is functioning", and the proposition l for "the light is on". Now, for instance, it holds that $\mathcal{M}, w_0 \models \neg[a][b]l$. So, agent a cannot ensure that agent b can ensure that the light is on. But also $\mathcal{M}, w_0 \models [a][b]\neg l$. So, agent a does have a possibility (namely, choosing λ_a) that ensures that subsequently, b can avoid l. Finally, we also have that $\mathcal{M}, w_0 \models [a][a,b]l$. That is, agent a can ensure (namely, choosing ρ_a) that the coalition $\{a, b\}$ can ensure that the light is on (namely, a choosing τ_a and b choosing λ_b or a choosing λ_a and b choosing τ_b).

3 Discrete STIT logic

The semantics of STIT is embedded in the branching time framework. It is based on structures of the form $\langle W, < \rangle$, in which W is a nonempty set of moments, and $<$ is a tree-like ordering of these moments, such that for any w_1, w_2 and w_3 in W, if $w_1 < w_3$ and $w_2 < w_3$, then either $w_1 = w_2$ or $w_1 < w_2$ or $w_2 < w_1$. Moreover, we here constrain the $<$ to be a *discrete* ordering, that is to say that, given a moment w_1, there exists a (not necessarily unique) successor moment w_2 such that $w_1 < w_2$ and there is no moment w_3 such that $w_1 < w_3 < w_2$.

A maximal set of linearly ordered moments from W is a *history*. Thus, $m \in h$ denotes that the moment m is *on* the history h. We define $Hist$ as the set of all histories of a STIT structure. $H_w = \{h | h \in Hist, w \in h\}$ denotes the set of histories passing through w. An *index* is a pair w/h, consisting of a moment w and a history h from H_w (i.e. a history and a moment in that history).

A *STIT model* is a tuple $\mathcal{M} = \langle W, Choice, <, v\rangle$, where:

- $\langle W, <\rangle$ is a branching-time structure.
- $Choice : \mathcal{A}gt \times W \mapsto 2^{2^{Hist}}$ is a function mapping each agent and each moment w into a partition of H_w. The equivalence classes belonging to $Choice_a^w$ can be thought of as possible choices or actions available to a at w. Given a history h, $Choice_a^w(h)$ represents the particular choice from $Choice_a^w$ containing h, or in other words, the particular action performed by a at the index w/h. We must have $Choice_a^w \neq \emptyset$ and $Q \neq \emptyset$ for every $Q \in Choice_a^w$.
- v is valuation function $v : \mathcal{A}tm \mapsto 2^{W \times Hist}$.

In STIT models, moments may have different valuations, depending on the history they are living in (cf. [14, footnote 2 p. 586]). Thus, at any specific moment, we have different valuations corresponding to the results of the different (non-deterministic) actions possibly taken at that moment.

In order to deal with group agency, Horty defines in [13, Section 2.4], the notion of collective choice. Horty first introduces action selection functions s_w from $\mathcal{A}gt$ into 2^{H_w} satisfying the condition that for each $w \in W$ and $a \in \mathcal{A}gt$, $s_w(a) \in Choice_a^w$. So, a selection function s_w selects a particular action for each agent at w.

Then, for a given w, $Select_w$ is the set of all selection functions s_w. For every $s_w \in Select_w$, it is assumed that $\bigcap_{a\in\mathcal{A}gt} s_w(a) \neq \emptyset$. This constraint corresponds to the hypothesis that the agents' choices are independent, in the sense that agents can never be deprived of choices due to the choices made by other agents.[5] Moreover, in order to match CL, we assume that $\bigcap_{a\in\mathcal{A}gt} s_w(a)$ is exactly $H_{w'}$ of a next moment w'. As explained in [8], this determinism is not a limitation of the modelling capabilities of the langauge, since we could introduce a neutral agent "nature", in order to accommodate non-deterministic transitions.

[5] Note that from this constraint it follows that two agents cannot possibly have an identical set of choices at the same moment. It also follows that there are not less than $\prod_{a\in\mathcal{A}gt} |Choice_a^w|$ histories passing through a moment w. Moreover, at moments where the minimal number of histories satisfies this constraint, choices at future moments will be vacuous and deterministic.

Using choice functions s_w, the $Choice$ function can be generalized to apply to groups of agents ($Choice : 2^{\mathcal{A}gt} \times W \mapsto 2^{2^{Hist}}$). A collective choice for a group of agents $A \subseteq \mathcal{A}gt$ is defined as:

$$Choice^w_A = \{\bigcap_{a \in A} s_w(a) | s_w \in Select_w\}$$

Semantics

Any STIT formula φ is evaluated with respect to a model $\mathcal{M}$ and an index w/h.

$$\mathcal{M}, w/h \models p \iff w/h \in v(p), p \in \mathcal{A}tm.$$

$$\mathcal{M}, w/h \models \neg\varphi \iff \mathcal{M}, w/h \not\models \varphi$$

$$\mathcal{M}, w/h \models \varphi \vee \psi \iff \mathcal{M}, w/h \models \varphi \text{ or } \mathcal{M}, w/h \models \psi$$

Historical necessity (or inevitability) at a moment w in a history is defined as truth in all histories passing through w:

$\mathcal{M}, w/h \models \Box\varphi \iff \mathcal{M}, w/h' \models \varphi, \forall h' \in H_w$.

When $\Box\varphi$ holds at w then φ is said to be settled true at w. $\Diamond\varphi$ is defined in the usual way as $\neg\Box\neg\varphi$, and stands for historical possibility.

There are several STIT operators; we here just introduce the so-called Chellas' STIT which is defined as follows:

$\mathcal{M}, w/h \models [A\, cstit\colon \varphi] \iff \mathcal{M}, w/h' \models \varphi, \forall h' \in Choice^w_A(h)$.

Intuitively it means that group A's current choices ensure φ, whatever other agents outside A do.

As we have discrete time, we can also define the temporal next ($\mathbf{X}$) operator:

$\mathcal{M}, w/h \models \mathbf{X}\varphi \iff \exists w' \in h\,(w < w', \mathcal{M}, w'/h \models \varphi, \nexists w'' \in h\,(w < w'' < w'))$.

The next operator is not standard in STIT formalisms, but we will need it to account for a translation of the nesting of coalition modalities from CL.

We write $\models_{STIT} \varphi$, if for every STIT model $\mathcal{M}$, every h in $\mathcal{M}$ and every moment w in h we have $\mathcal{M}, w/h \models \varphi$.

As shown in [14], both Chellas' STIT and historical necessity are S5 modal operators, and $\models_{STIT} \Box\varphi \rightarrow [A\, cstit\colon \varphi]$.

Figure 2 is an example of a STIT model. A feature of this model that does not hold for STIT models in general, is that all indexes m/h for a moment m have the same valuation of atomic propositions. This is done here to stress the relation with the CL model of Figure 1, and preludes the translation we give in the next section. For any history h through w_0 we have $\mathcal{M}, w_0/h \models \neg\Diamond[a\, cstit\colon$

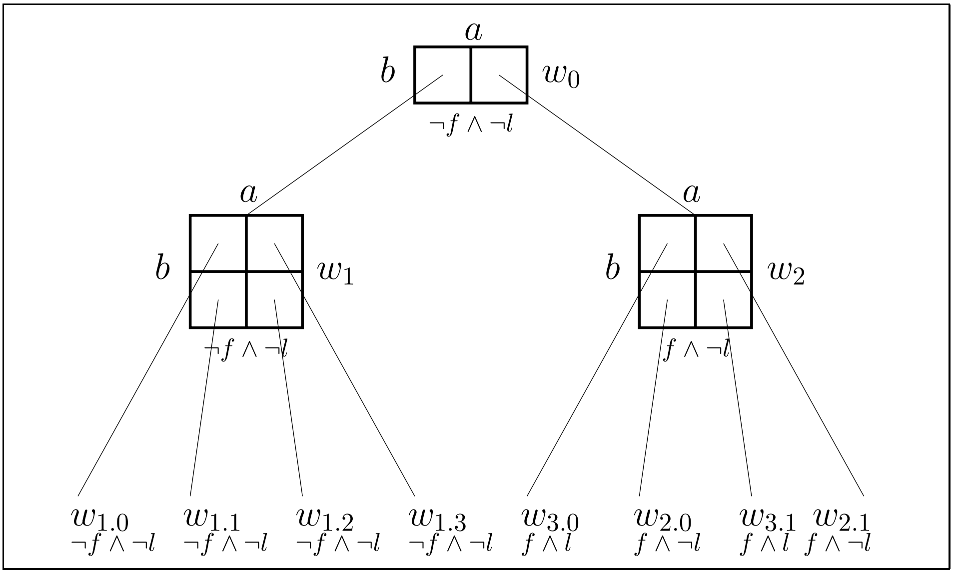

Fig. 2. Example of STIT model.

$\mathbf{X}\Diamond[b\, cstit: \mathbf{X}l]]$. Also we have $\mathcal{M}, w_0/h \models \Diamond[a\, cstit: \mathbf{X}\Diamond[b\, cstit: \mathbf{X}\neg l]]$ and $\mathcal{M}, w_0/h \models \Diamond[a\, cstit: \mathbf{X}\Diamond[\{a,b\}\, cstit: \mathbf{X}l]]$, analogous to the properties we had in the CL model.

4 From Coalition Logic to STIT logic

The structural similarities between the formulas interpreted over the CL model of Figure 1 and the formulas interpreted over the STIT model of Figure 2, suggest the translation that is formalized below.

We define the translation tr from CL formulae to $STIT$ formulae as:

$$\begin{aligned} tr(p) &= \Box p, \text{ for } p \in \mathcal{A}tm \\ tr(\neg\varphi) &= \neg tr(\varphi) \\ tr(\varphi \vee \psi) &= tr(\varphi) \vee tr(\psi) \\ tr([A]\varphi) &= \Diamond[A\, cstit: \mathbf{X} tr(\varphi)] \end{aligned}$$

Note that $\models_{STIT} tr(\varphi) \equiv \Box tr(\varphi)$. (The proof uses the fact that the logic of historical necessity $\Box$ is S5.)

Theorem 4.1 *If φ is CL-satisfiable then $tr(\varphi)$ is STIT-satisfiable.*

Proof. For any game model $\mathcal{M}_{CL} = \langle W_{CL}, \{\Sigma_{a,w} | a \in \mathcal{A}gt, w \in W_{CL}\}, o, v_{CL}\rangle$, we define $\mathcal{M}'_{CL} = \langle T_{CL}, \{\Sigma_{a,w} | a \in \mathcal{A}gt, w \in T_{CL}\}, o, v_{CL}\rangle$ to be the game model that results from unravelling the function o into a tree. (Thus, $W_{CL} \subseteq$

T_{CL}, where the possible difference between these sets are *semantically indistinguishable duplicates* of worlds in W_{CL}.) From similar results in monotone modal logic [10] and normal modal logic [4], it is immediately clear that the unravelled model is satisfiable if the original model is. The second step is to associate with every game-model $\mathcal{M}'_{CL} = \langle T_{CL}, \{\Sigma_{a,w} | a \in \mathcal{A}gt, w \in T_{CL}\}, o, v_{CL}\rangle$, a STIT model $\mathcal{M}'_{STIT} = \langle W_{STIT}, Choice, <, v_{STIT}\rangle$, satisfying the following conditions:

- $W_{STIT} = T_{CL}$
- $w < w' \iff \exists u_1, \ldots, u_n \, (u_1 = w, u_n = w', \forall i < n \, (\exists \sigma_{\mathcal{A}gt,u_i} \, (o(\sigma_{\mathcal{A}gt,u_i}) = u_{i+1})))$
- $\forall a, \forall w, Choice^w_a = \{\{h \in Hist \mid h \cap o(\sigma_{a,w}) \neq \emptyset\} | \sigma_{a,w} \in \Sigma_{a,w}\}$
  Thus, every element of $Choice^w_a$ collects the histories (recall that these are sets of states) passing through the outcomes $o(\sigma_{a,w})$ for some action $\sigma_{a,w}$.
- $\forall w, \forall h \in H_w, v_{STIT}(w/h) = v_{CL}(w)$

It is straightforward to check that $\mathcal{M}'_{STIT}$ is indeed a discrete STIT model, and that for any game tree model there is always exactly one such an associated model.

We now prove that $\mathcal{M}'_{STIT}$ satisfies a translated formula if the game tree model it is associated to satisfies the original formula. That is, we prove (by structural induction on φ) that $\mathcal{M}'_{CL}, w \models \varphi$ only if $\mathcal{M}'_{STIT}, w/h \models tr(\varphi), \forall h \in H_w$. Cases of atomic formulae, negations and disjunctions are trivial, and we here only present the case where $\varphi = [A]\psi$. $\mathcal{M}'_{CL}, w \models [A]\psi$ means that there exists a $\sigma_{A,w}$ such that for all $u \in o(\sigma_{A,w})$ we have $\mathcal{M}'_{CL}, u \models \psi$. So by induction hypothesis, for all $u \in o(\sigma_{A,w})$ and for all $h \in H_u$, $\mathcal{M}'_{STIT}, u/h \models tr(\psi)$. By construction of $Choice^w_A$, this is true only if there is a partition choice $Q \in Choice^w_A$ such that for all histories $h \in Q$ we have $\mathcal{M}'_{STIT}, u/h \models tr(\psi)$. By construction of $<$, this means that $\mathcal{M}'_{STIT}, w/h \models \mathbf{X}tr(\psi)$. We also can deduce that for all $h \in Q$ we have $\mathcal{M}'_{STIT}, w/h \models [A\, cstit: \mathbf{X}tr(\psi)]$. And then for all $h \in H_w$ we have $\mathcal{M}'_{STIT}, w/h \models \Diamond[A\, cstit: \mathbf{X}tr(\psi)]$. □

Theorem 4.2 *If* $\models_{CL} \varphi$ *then* $\models_{STIT} tr(\varphi)$.

Proof. Instead of a semantical proof, we use the axiomatization of [17]: we prove that the translations of the axioms are valid, and that the translated inference rules preserve validity.

It is obvious that the translation of (RE) preserves validity.

The translation of axiom ($\perp$) is $\neg\Diamond[A\, cstit: \mathbf{X}\perp]$, which is equivalent to $\Box\neg[A\, cstit: \mathbf{X}\perp]$. To see that this is valid note first that $\models_{STIT} \neg[A\, cstit: \perp]$ because each element of $Choice^w_a$ is nonempty, for any a and w. From the latter it follows that $\models_{STIT} \neg[A\, cstit: \mathbf{X}\bot]$ (because $\models_{STIT} \bot \equiv \mathbf{X}\bot$, and because the STIT logic satisfies the rule of substitution of equivalences). Then by the necessitation rule for $\Box$ (which is valid in STIT models) we get $\models_{STIT} \Box\neg[A\, cstit: \mathbf{X}\bot]$.

The translation of axiom ($\top$) is $\Diamond[A\, cstit: \mathbf{X}\top]$. First, $\models_{STIT} [A\, cstit: \top]$ because the rule of necessitation holds for $[A\, cstit: \_]$. Second, $\models_{STIT} [A\, cstit: \mathbf{X}\top]$ (because $\top \equiv \mathbf{X}\top$, and because STIT satisfies the rule of substitution of equivalences). Third, $\models_{STIT} \Diamond[A\, cstit: \mathbf{X}\top]$ because $\models_{STIT} \psi \to \Diamond\psi$ holds for any ψ (due to the T-axiom for $\Box$).

The translation of (N) is $\neg\Diamond[\emptyset\, cstit: \mathbf{X}\neg tr(\varphi)] \to \Diamond[\mathcal{A}gt\, cstit: \mathbf{X}tr(\varphi)]$. This is valid in STIT because $\models_{STIT} \mathbf{X}\neg\psi \equiv \neg\mathbf{X}\psi$ and $\models_{STIT} \neg[\emptyset\, cstit: \neg\psi] \to [\mathcal{A}gt\, cstit: \psi]$ for all ψ, due to our supplementary condition that $Choice^w_{\mathcal{A}gt}$ must be a singleton.

The translation of (M) is $\Diamond[A\, cstit: \mathbf{X}(tr(\varphi) \wedge tr(\psi))] \to \Diamond[A\, cstit: \mathbf{X}tr(\varphi)]$. This is STIT-valid first because $\mathbf{X}$ is a normal modal operators, i.e. $\models_{STIT} \mathbf{X}(tr(\varphi) \wedge tr(\psi)) \to \mathbf{X}tr(\varphi)$. And second because $[A\, cstit: \_]$ and $\Box$ are also normal modal operator: from $\gamma_1 \to \gamma_2$ we can infer $[A\, cstit: \gamma_1] \to [A\, cstit: \gamma_2]$ and $\Diamond[A\, cstit: \gamma_1] \to \Diamond[A\, cstit: \gamma_2]$.

The translation of axiom (S) is $\Diamond[A_1\, cstit: \mathbf{X}tr(\varphi)] \wedge \Diamond[A_2\, cstit: \mathbf{X}tr(\psi)] \to \Diamond[A_1 \cup A_2\, cstit: \mathbf{X}(tr(\varphi) \wedge tr(\psi))]$, for $A_1 \cap A_2 = \emptyset$. This is valid in STIT because the choices of a group A in STIT models are constructed just as the outcome function of A in CL, viz. by pointwise intersection. □

Theorem 4.3 *φ is satisfiable in CL iff $tr(\varphi)$ is satisfiable in STIT.*

Proof. This is an immediate corollary of theorems 4.1 and 4.2. □

5 Discussion

As we have said, our translation requires the addition of two constraints to the *vanilla* STIT:

(i) $<$ must be discrete

(ii) $\forall w \in W, \exists w' \in W\, (w < w'$ and $\nexists w'' \in W, w < w'' < w', \bigcap_{a\in\mathcal{A}gt} s_w(w) = H_{w'})$

Intersection of all agents' choices is *not only* nonempty but must exactly be the set of histories passing through a next moment.

The first constraint permits us to define the $\mathbf{X}$ operator, which in the STIT context accounts for the notion of *outcome*. Remark that the introduction of the next operator is essential. The reason is that the coalition operators of CL may be nested. If we want to express this nesting of operators in the

STIT context we must be able a facility to move to the next moment. Thus a translation that does not use an X cannot work. For instance, the one we get by replacing the final clause by $tr2([A]\varphi) = \Diamond[A\,cstit\colon tr2(\varphi)])$, does not work because it runs into trouble if we translate nested coalition formulas like $[A](p \wedge \neg[A]p)$. However, we have to be careful where to insert the X operator in the translation. For instance, the translation we get by replacing the final clause by $tr3([A]\varphi) = \Diamond X[A\,cstit\colon tr3(\varphi)]$ does not work because it leads to violation of the super additivity principle (S).

The second constraint is a direct translation of the CL constraint stating that when every agent in $\mathcal{A}gt$ opts for an action, the next state of the world is completely determined. In STIT this amounts to defining that the intersection of $\mathcal{A}gt$'s choices must be exactly the set of histories passing through this moment.

Given these extra constraints, the idea behind our translation tr, is that CL models match with discrete STIT models, where atomic propositions are historically necessary. Clearly, CL can not support different valuations of the same atom at one state.

The translation thus depends on the historic necessity of atomic propositions in STIT worlds. However, a satisfiability preserving translation without the clause $tr(p) = \Box p$, for $p \in \mathcal{A}tm$ is also possible.

$$
\begin{aligned}
tr4(p) &= p, \text{ for } p \in \mathcal{A}tm\\
tr4(\neg\varphi) &= \neg tr4(\varphi)\\
tr4(\varphi \vee \psi) &= tr4(\varphi) \vee tr4(\psi)\\
tr4([A]\varphi) &= X\Diamond[A\,cstit\colon tr4(\varphi)]
\end{aligned}
$$

A slightly more elaborate proof that this translation works can be made along the same lines.

Finally we mention that the translations are close to *simulations* of the weak modal operator $[A]\varphi$ in terms of the two normal S5 modal operators $\Diamond\varphi$ and $[A\,cstit\colon \varphi]$. Similar simulations have been given for weak modal logics [7,6]. Another simulation of the CL operator $[A]\varphi$ was recently given by van der Hoek and Wooldridge [12]. For reasoning about *propositional control*, they simulate (although they do not use this terminology) $[A]\varphi$ by $\Diamond_A\Box_{\overline{A}}\varphi$, where the diamond and the box are normal modal operators.

6 Conclusion

We have established that Coalition Logic can be embedded in STIT logic. CL is a fragment of Alternating-time Temporal Logic. Therefore, it would be interesting to investigate translations from ATL to STIT. We believe that this can be done, by introducing strategies into the STIT framework as done in [3,13].

A more challenging research avenue is to import deontic concepts that have been investigated in the STIT framework such as in [14,13] into CL and ATL. It seems that this can be done in a rather straightforward manner.

Already based on our translation, it has been shown in [11] that the problem of uniform strategies with imperfect knowledge (devised in [15]) can be solved in STIT.

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