Careful rational synthesis in games with multiple common resources

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ABSTRACT

Careful rational synthesis was defined in previous work as a quantitative extension of Fisman et al.'s rational synthesis. Agents are interacting in a graph arena in a turn-based fashion, there is one common resource, and each action may decrease or increase the resource. Each agent has a temporal qualitative objective and wants to maintain the value of the resource positive. One must find a Nash equilibrium. This problem is decidable.

In more practical settings, the verification of the critical properties of multiagent systems calls for models with many resources. Indeed, agents and robots consume and produce more than one type of resources: electric energy, fuel, raw material, manufactured goods, etc. We thus explore the problem of careful rational synthesis with several resources. We show that the problem is undecidable. We then propose a variant with bounded resources, motivated by the observation than in practical settings, the storage of resources is limited. We show that the problem becomes decidable, and is no harder than plain controller synthesis with Linear-time Temporal Logic.

KEYWORDS

Rational Synthesis, Temporal Logic, Resources

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1 INTRODUCTION

The presence of autonomous agents in modern societies has become commonplace. We interact with them everyday, and they may be of different levels of autonomy, e.g., self-checkout, chatbots, robot vacuum cleaners, or virtual assistants. A current tendency is that agents are intruding on the physical world, and robots are expanding their territory beyond their confined industrial environment.

The access to the resources necessary for an agent to accomplish his tasks could have been simply assumed in many application domains before: direct wire to an electricity source, a human operator providing raw material, etc. Nowadays, typical agents must be more autonomous than before in managing the multiple resources they need. They must carefully consume them, and in presence of competitors, they must also be careful in how they produce them.

Linear-time Temporal Logic (LTL) [16] has been a very popular logic for specifying temporal properties of systems. Planning with objectives expressed in some temporal logic has been well studied [3, 4, 6, 9]. Some logics have also been proposed to explicitly verify the properties of multiagent systems in presence of resource constraints [2, 5, 15]. When agents roam more freely the physical world, they are more likely to compete with other agents, human or artificial, which may have conflicting goals. When planning in such environments an agent needs to adapt his behaviour to the capabilities and goals of others. A solution to a multiagent planning problem in this setting is a non-cooperative strategic equilibrium: a vector of strategies, one for each agent, such that no individual agent can be better off by unilaterally changing their strategy. This is what has come to be known as a Nash equilibrium [14].

This paper aims to contribute to the line of research interested in the formal verification of the existence of Nash equilibria in a multiagent system [1, 8, 12, 18]. When there is a solution Nash equilibrium, the techniques used can actually return a multiagent plan that satisfies the requisites. The paper has the special focus to consider agents that must be autonomous in an environment with multiple common resources, so as to bring the theory closer to the reality that engineers are working with.

In [7], the problem of careful rational synthesis is defined as a quantitative variant of rational synthesis [12]. Agents interact in a graph arena in a turn-based fashion. Each state is controlled by one and only one agent who decides which edge to follow. Each

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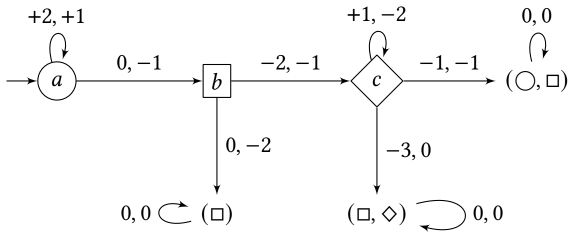

Figure 1: A 2-resource 3-player game.

agent has a temporal objective that he tries to achieve. There is one integer common resource, and each action may decrease or increase the resource. The rational synthesis problem consists in computing a Nash equilibrium that satisfies a system objective. It is shown that in presence of one common resource, deciding the existence of a strategic equilibrium for careful autonomous agents (with *parity* objectives, a canonical representation of temporal properties on infinite traces [10]) can be solved in polynomial space. With LTL objectives, the problem can be solved in doubly exponential space.

But in real-case scenarios, physical agents are operating in a world where there is more than one resource.

2 CAREFUL RATIONAL SYNTHESIS WITH SEVERAL RESOURCES – GENERAL CASE

In this paper, we explore the problem of careful rational synthesis in the commons with several resources.

Example 2.1. Consider the game illustrated on Figure 1. Players 1, 2 and 3 control the states a, b, and c respectively. Player 1 wants to reach a state with ○, Player 2 wants the reach a state with □, Player 3 wants to reach a state with ◇. All of them want to keep the resources in check: they would be dissatisfied if any of the resources were to go below zero. The objective of the system is ○. A solution to the synthesis problem is thus a Nash equilibrium that reaches the state (○, □), and never depletes the resources.

One starts with the resources being $(0, 0)$. Player 1 must pump thrice on a, which brings the resources to $(6, 3)$. (Only he can increase resource two, and at least an amount of 3 is necessary to reach his objective and the objective of the system.) Player 1 can then go to b, which brings the resources to $(6, 2)$.

At that point, Player 2 could go down. This would be the outcome of a Nash equilibrium, but it would not be a solution to our synthesis problem since we are seeking an equilibrium satisfying the system's objective. Instead, let Player 2 go to c; this brings the resources vector to $(4, 1)$.

At that point, Player 3 can go down. Once again this is the outcome of a Nash equilibrium, but this would not be solution. Instead, Player 3 could go right, and the run so obtained would satisfy the objective of the system and keep the resources in check. However this is not the outcome of a Nash equilibrium since Player 3 can deviate and increase his payoff by going down.

In fact, there is no solution to the careful rational synthesis problem.

It is unfortunately a negative result that we must report. Deciding the existence of a strategic equilibrium for careful autonomous agents in environments with multiple common resources is indeed undecidable. To obtain the result we proceed in two steps. We introduce a variant of *multi-counter automata* as a natural generalisation of *bounded one-counter automata* from [11]. We show that the problem of reachability in multi-counter automata is undecidable through a reduction from the problem of reachability in two-counter Minsky machines [13]. The reduction[1] presented in [7, Sec. 4.2] is then adapted to obtain a reduction from the problem of reachability in multi-counter automata into the problem of careful rational synthesis in the commons with several resources.

Theorem 2.2. *The careful rational synthesis in the commons is undecidable, even with two players and two resources, and with reachability objectives.*

3 CASE WITH BOUNDED RESOURCES

We then propose a variant with bounded resources. In this setting, every resource has a maximum capacity.

Example 3.1. In the game illustrated on Figure 1, suppose now that both resources are bounded with bounds $(3, 3)$.

As before, Player 1 must pump thrice on a, with the resources values being successively $(2, 1)$, $(3, 2)$, and $(3, 3)$. As before, Player 2 could win going down, and again this would be the outcome of a Nash equilibrium but would not be a solution. Instead, let Player 2 go to c, which brings the resources to $(1, 2)$.

To be happy Player 3 must go down. To not bring the resource one below zero, he must pump on c twice to bring the value of the first resource to 3, but doing so would deplete the second resource. If Player 3 instead carefully takes the play to the right, Player 1 and Player 2 meet their objectives, and so does the system. Hence this outcome results in a Nash equilibrium, that is a solution.

To summarize, when the resources are bounded with bounds $(3, 3)$, the strategies of Player 1 taking the self-loop on a thrice, then going to b; Player 2 going to c; and Player 3 going to (○, □) form a Nash equilibrium, and is a solution to the careful rational synthesis problem.

This variant with bounded resource storage capacity is of interest for the practical engineering of autonomous multiagent systems for two reasons. The first reason is conceptual. In many real-case scenarios, resources are bounded: e.g., in a community, a shared tank of water can only contain a predetermined amount of water, a shared microgrid powerpack can only contain a predetermined amount of energy, etc.

The second reason is algorithmic. We show that unlike in the setting with unbounded resources, the problem of rational synthesis in this bounded setting becomes decidable. Even better, with objectives expressed in LTL, it is not harder than the plain reactive synthesis problem, which is 2EXPTIME-complete [17]. The result is obtained through an appropriate unfolding of the game arena into an arena without costs. The rational synthesis with LTL objectives from [12] can then be applied at once.

[1] The reduction in [7, Sec. 4.2] is from reachability in bounded one-counter automata to careful rational synthesis in the commons with one resource. It serves to established PSPACE-hardness of the problem of careful rational synthesis with parity objectives, which is PSPACE-complete.

Theorem 3.2. *The careful rational synthesis in the commons with several bounded resources is* 2EXPTIME*-complete.*

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