

Energy sensitive rational synthesis*

Rodica Condurache

Computer Science Department,
“A.I.Cuza” University, Iași, ROMANIA
rodica.b.condurache@gmail.com

Catalin Dima

Université Paris Est Créteil, LACL(EA 4219),
UPEC 94010 Créteil Cedex, France
dima@u-pec.fr

Youssef Oualhadj

youssouf.oualhadj@lacl.fr

Nicolas Troquard

The KRDB Research Centre, Free University of Bozen-Bolzano
I-39100 Bozen-Bolzano BZ, Italy
nicolas.troquard@unibz.it

We study the rational synthesis problem considering that the interaction of agents imply the consumption of some shared resource/energy. We consider the problem in cooperative and non-cooperative setting combined with careless or careful agents.

1 Introduction

Common-pool resources are those resources like water, air, coal, pastures, or fish stocks [18]. They are *non-excludable*: they are out there for the taking. They are *rivalrous*: one player’s consumption can limit or prevent another player to consume it. *Energy* also is a kind of resource that typically can be framed as a commons [17]. In this paper, we will focus on the resource of energy, as it is understood in the literature in Computer Science. In fact, it is both more abstract, and much more than ‘energy’: it captures any common-pool resource that can be conveniently quantified by assigning a number to it, and where a bigger number indicates a greater amount of the resource.

In this paper, the agents interact in a turn-based graph arena, where in each state, one player decides which edge to follow. To each edge is associated an integer which corresponds to the *energy cost* incurred by the whole system when it follows this edge. Each player has a temporal objective, expressed as an LTL formula over the set of states.

For a long time, it was believed that common-pool resources were bound to collapse due to the actions of self-interested players, causing the resources to be depleted or spoiled. The great contribution of E. Ostrom [18] was to evidence wide-ranging instances where this is not in fact the case, and to identify design principles of successful common-pool resource management. The models and algorithmic solutions presented here are but a theoretical contribution. We can hope that pursued efforts in controller synthesis for resource-sensitive multi-agent systems may yield useful engineering solutions for future commons management.

We study *cooperative* synthesis problem [11]. In this problem, agent 0 holds the special role of *controller*. Cooperative synthesis consists in finding a strategy for agent 0 such that in the sub-game that it defines, *there exists* a Nash Equilibrium whose outcome satisfies agent 0’s objective, and never depletes the system’s resources. For non cooperative synthesis, this must be the case for *all* Nash Equilibria in the sub-game. In other words the specification of agent 0 has to hold in *sustainable* manner in every rational behavior of the other agents. We will consider two types of agents: *careful* and *careless*. Careless agents bother only for their objective. Careful agents also pay attention to not deplete the system’s resources. (Agent 0 is always careful.)

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2 Preliminaries

2.1 Turn based games

A *turn based arena* is a tuple $\mathcal{G} = \langle \text{St}, (\text{St}_0 \uplus \dots \uplus \text{St}_n), s_0, \text{Agt}, \text{Edge} \rangle$, where St is a finite set of states, $(\text{St}_0 \uplus \dots \uplus \text{St}_n)$ is a partition of St , s_0 is an initial state $\text{Agt} = \{0, \dots, n\}$ is the set of agents s.t. agent i controls the states in St_i , and Edge is in an edge relation in $\text{St} \times \text{St}$.

Throughout the paper we use the following notations. If Σ is a finite set, then Σ^* is the set of finite sequences of elements of Σ , usually called finite words. Σ^ω is the set of infinite sequences of elements of Σ , usually called infinite words. For every word (finite or infinite) w in Σ^ω and every $i \geq 0$, we denote by $w[i]$ the $i+1$ -th letter in w ; we denote by $w[..i]$ the prefix of w of size $i+1$ and $w[i..]$ the suffix that starts at the $i+1$ -th letter. For any finite non empty word w , we define $\text{Last}(w)$ as the last letter of w , and $|w|$ the length of w . A *play* in an arena \mathcal{G} is an infinite word $\rho = s_0 s_1 s_2 \dots$ in St^ω such that s_0 is the initial state and for all $i \geq 0$, $(s_i, s_{i+1}) \in \text{Edge}$. $\text{Plays}(\mathcal{G})$ is the set of all the plays in the arena \mathcal{G} . We call any finite word in St^* that is prefix of an element in $\text{Plays}(\mathcal{G})$ a *history*, $\text{Hist}(\mathcal{G})$ is the set of all the histories of \mathcal{G} . We also define $\text{Hist}_i(\mathcal{G})$ for i in Agt as follows:

$$\text{Hist}_i(\mathcal{G}) = \{h \in \text{Hist}(\mathcal{G}) \mid \text{Last}(h) \in \text{St}_i\} .$$

A *strategy* for agent i is a mapping $\sigma_i : \text{Hist}_i(\mathcal{G}) \rightarrow \text{St}$. Once we fixed a strategy for each agent, we obtain a strategy profile. Formally, a *profile of strategies* is defined as a tuple of strategies $\bar{\sigma} = \langle \sigma_0, \sigma_1, \dots, \sigma_n \rangle$. We will sometimes economically write *profile* instead of profile of strategy. For $0 \leq i \leq n$, we denote $\bar{\sigma}[i]$ the strategy at the $i+1$ -th position (i.e., of agent i), and by $\bar{\sigma}_{-i}$ we mean the tuple obtained from $\bar{\sigma}$ by removing the strategy $\bar{\sigma}[i]$ also called *partial profile*. Finally, the tuple of strategies $\langle \bar{\sigma}_{-i}, \sigma'_i \rangle$ is obtained from the profile $\bar{\sigma}$ by substituting agent i 's strategy with σ'_i . Once a profile is chosen it induces a unique play ρ , that we call the outcome of $\bar{\sigma}$, and we denote it $\text{Out}(\bar{\sigma})$. We say that a play $\rho = s_0 s_1 s_2 \dots$ is *compatible* with a strategy $\bar{\sigma}[i]$ of agent i if $\rho[0] = s_0$, and $\forall k \geq 0$, $\bar{\sigma}[i](\rho[..k]) = \rho[k+1]$. Since $\text{Out}(\bar{\sigma})$ is unique we will confound $\bar{\sigma}$ and $\text{Out}(\bar{\sigma})$ in the sequel unless unclear from the context.

A *cost arena*, is any tuple $\langle \mathcal{G}, \text{Cost} \rangle$ where \mathcal{G} is a turn based arena, and $\text{Cost} : \text{Edge} \rightarrow \mathbb{Z}$ is a cost function. Let $h = s_0 s_1 \dots s_n s_{n+1}$ be a history in $\text{Hist}(\mathcal{G})$, we abusively write $\text{Cost}(h)$ to mean the extension of Cost to histories that is $\text{Cost}(h) = \sum_{i=0}^n \text{Cost}(s_i, s_{i+1})$. We denote W the largest absolute value that appears in Cost . We also mention that we consider that values of Cost are encoded in binary, thus W is exponential in its encoding which is of size $\log(W)$.

2.2 Specification, payoffs and solution concepts

A specification for an agent is a subset of St^ω . For every i in Agt , we denote Spec_i its specification. Once we define Spec_i for every i in Agt , then $\langle \mathcal{G}, \text{Cost}, \text{Spec}_0, \dots, \text{Spec}_n \rangle$ is called a cost game (or simply game). The payoff of each agent in this game is $\text{Payoff}_i : \rho \mapsto 1$ iff $\rho \in \text{Spec}_i$ and 0 otherwise.

We say that the play ρ is *feasible* if for every $k \geq 1$ we have $\text{Cost}(\rho[..k]) \geq 0$, i.e., every history of ρ induces a non negative cost. We denote the set of feasible plays of \mathcal{G} by $\text{Feas}(\mathcal{G})$. Let $\bar{\sigma}$ be a profile, the payoff of $\bar{\sigma}$ for agent i is $\text{Payoff}_i(\text{Out}(\bar{\sigma}))$. When clear from the context we simply write $\text{Payoff}_i(\bar{\sigma})$. We say that $\bar{\sigma}$ is feasible if $\text{Out}(\bar{\sigma})$ is in $\text{Feas}(\mathcal{G})$. We sometimes abusively write $\bar{\sigma} \in \text{Feas}(\mathcal{G})$ instead of $\text{Out}(\bar{\sigma}) \in \text{Feas}(\mathcal{G})$.

We will consider Nash equilibria as a concept of solution. A Nash equilibrium is defined as follows:

Definition 1. Let $\bar{\sigma}$ be a profile, $\bar{\sigma}$ is a Nash equilibrium (NE) if for every agent i and every strategy σ'_i of i the following holds true: $\text{Payoff}_i(\bar{\sigma}) \geq \text{Payoff}_i(\langle \bar{\sigma}_{-i}, \sigma'_i \rangle)$.

Throughout this paper, we will assume that agent 0 is the agent for whom we wish to synthesize the strategy, therefore, we use the concept of 0-fixed Nash equilibria (0-NE).

Definition 2. A profile $\langle \sigma_0, \bar{\sigma}_0 \rangle$ is a 0-NE, if for every strategy σ_i for agent i in $\text{Agt} \setminus \{0\}$ the following holds true: $\text{Payoff}_i(\langle \sigma_0, \bar{\sigma}_0 \rangle) \geq \text{Payoff}_i(\langle \sigma_0, \langle (\bar{\sigma}_0)_{-i}, \sigma_i \rangle \rangle)$.

That is, by fixing the strategy σ_0 for agent 0, the other agents cannot improve their payoff by unilaterally changing their strategy. We denote by 0-NE(\mathcal{G}) the set of all the 0-NE of \mathcal{G} .

2.3 Careless and careful specifications and rationality

For each agent i , the specifications Spec_i (as thus payoff function Payoff_i) will depend on a qualitative objective; and it may also depend on a quantitative objective (agent 0's specification always does). Whether the specifications depends or not a quantitative objective will yield careful and careless specifications.

In this paper we consider *LTL* qualitative specifications, i.e., each player in Agt is assigned an LTL formula ϕ_i that represents a temporal objective they want to achieve.

The formula $X\phi$ holds true on ρ if ϕ is true next. The formula $\phi U \psi$ holds true on ρ if ϕ is true at least until ψ is true. We use $\diamond\phi$ as shorthand for $\top U \phi$, i.e., eventually ϕ is true, and $\square\phi$ as a shorthand for $\neg\diamond\neg\phi$, i.e., ϕ is always true. Let ρ be a play in $\text{Plays}(\mathcal{G})$, if it satisfies a formula ϕ we write $\rho \models_{\mathcal{G}} \phi$. We sometime drop the subscript \mathcal{G} if clear from the context. Let i be an agent, we write Obj_i as the set $\{\rho \in \text{Plays}(\mathcal{G}) \mid \rho \models_{\mathcal{G}} \phi_i\}$.

Careless specification and rationality In this setting the specification of agent 0, Spec_0 , is defined by the set of plays that are in $\text{Obj}_0 \cap \text{Feas}(\mathcal{G})$. The specification for the other agents, Spec_i for i in $\text{Agt} \setminus \{0\}$, is Obj_i . Thus, the agents in $\text{Agt} \setminus \{0\}$ are indifferent about the feasibility of plays. In particular, when defining 0-NEs, they can profitably deviate even if the resulting play is not feasible. It is enough that the resulting play satisfies their objective while the current play does not. We consider these agents careless.

Careful specification and rationality In this setting the specifications Spec_i for agent i in Agt is the set plays that are in $\text{Obj}_i \cap \text{Feas}(\mathcal{G})$. Thus, the agents in $\text{Agt} \setminus \{0\}$ are concerned with the feasibility of plays. In particular, when defining 0-NEs, they can profitably deviate from a profile only if the resulting play is feasible. We consider these agents careful.

2.4 Decision problems

Now fix a game $\langle \mathcal{G}, \text{Cost}, \text{Spec}_0, \dots, \text{Spec}_n \rangle$, in both cases of careful and careless specifications, we want to solve the following problems.

Cooperative synthesis The first problem (cf. Pb. 3) is an extension of the cooperative rational synthesis introduced in [11] (cf. Pb. 4).

Problem 3 (Energy Cooperative Rational Synthesis). *Is there a strategy σ_0 , and a tuple $\langle \sigma_1, \dots, \sigma_n \rangle$, s.t.: $\langle \sigma_0, \dots, \sigma_n \rangle \in \text{Spec}_0 \cap 0\text{-NE}(\mathcal{G})$?*

A solution is careless if agents in $\text{Agt} \setminus \{0\}$ are considered careless. It is careful if they are considered careful. The original problem of cooperative rational synthesis was defined in [11] and it is a qualitative version of the previous problem:

Problem 4. *Is there a strategy σ_0 , and a tuple $\langle \sigma_1, \dots, \sigma_n \rangle$, s.t.: $\langle \sigma_0, \dots, \sigma_n \rangle \in \text{Obj}_0 \cap 0\text{-NE}(\mathcal{G})$?*

Non cooperative synthesis The non cooperative setting was introduced in [14, 15] for purely quantitative objectives.

Problem 5 (Energy Non-Cooperative Rational Synthesis). *Is there a strategy σ_0 , such that for any tuple $\langle \sigma_1, \dots, \sigma_n \rangle$, the following holds: $\langle \sigma_0, \dots, \sigma_n \rangle \in 0\text{-NE}(\mathcal{G}) \implies \langle \sigma_0, \dots, \sigma_n \rangle \in \text{Spec}_0$?*

We also recall the original version which again, is a qualitative version of the previous problem.

Problem 6. *Is there a strategy σ_0 , such that for any tuple $\langle \sigma_1, \dots, \sigma_n \rangle$, the following holds: $\langle \sigma_0, \dots, \sigma_n \rangle \in 0\text{-NE}(\mathcal{G}) \implies \langle \sigma_0, \dots, \sigma_n \rangle \in \text{Obj}_0$?*

Remark 7. *The careful and careless semantics in Pb. 4, and Pb. 6 coincide since the feasibility requirement is ignored.*

Example 8. *For now we focus on the cooperative version of the problem. Consider the arena depicted in Fig. 1. In this game, the agent 0 (circle) wants to reach state $(1, 1, 0)$, agent 1 wants to reach state $(0, 1, 0)$ or $(1, 1, 0)$, and agent 2 wants to reach $(0, 0, 1)$. Clearly from state a , agent 0 has to move the play to state b , but since the cost of this edge is $_1$ he has to take the self enough times. In fact if he takes the selfloop of state a 3 times then takes the edge toward state b . From there, agent 1 can move the play down or right towards c . Assume he chooses the latter option, the play now is in state c and the cost of the current history that led to c is 1. Now it is up to agent 2 to decide where the play will end; in a careless setting he has no incentive to take the right edge but takes the edge toward $(0, 0, 1)$, but this is not a solution since the agent 0 does not satisfy his specification. In the carefully setting, agent 2 cannot take this very edge since it costs -2 , therefore his only possible move is $(1, 1, 0)$. This is a solution, since agent 0 achieves his specification and 1 has no incentive to deviate because his payoff is also 1.*

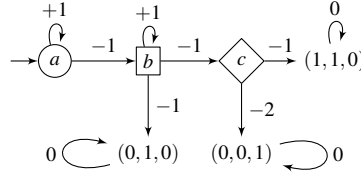


Figure 1: Game where there exists a careful solution, but no careless solution exists.

A zero-sum game $\langle \mathcal{G}, \text{Obj} \rangle$ is a turn based arena with two agents where $\text{Spec}_0 = \text{Obj} \subseteq \text{St}^\omega$ and $\text{Spec}_1 = \text{St}^\omega \setminus \text{Obj}$. We say that i has a winning strategy σ from some state s if for any strategy τ of $-i$, $\text{Out}(\sigma, \tau)$ is in Spec_i . When it is the case, we call s a winning state, and the set of winning states is called the winning region, denoted by $\text{Win}[\text{Spec}_i]$.

LTL synthesis Given an LTL formula ϕ defined on St , and a turn based game $\langle \mathcal{G}, \text{Obj} \rangle$ where $\text{Obj} = \{ \rho \in \text{Plays}(\mathcal{G}) \mid \rho \models \phi \}$, compute $\text{Win}[\text{Obj}]$. This problem is 2-EXPTIME-complete [19]. It follows from a reduction to a *zero-sum parity game* on an arena which is double exponential in the size of ϕ . A parity game is a zero-sum game where the objective is described as follows; for some priority function $\text{Pr} : \text{St} \rightarrow \mathbb{N}$, $\text{Parity} = \{ \rho \in \text{St}^\omega \mid \min\{\text{Pr}(s) \mid s \in \text{Inf}(\rho)\} \text{ is even} \}$, where $\text{Inf}(\rho)$ is the set of states occurring infinitely often along ρ , i.e., $\text{Inf}(\rho) = \{ s \in \text{St} \mid \forall i \geq 0, \exists j > i \text{ s.t. } \rho[j] = s \}$. Computing the winning region in a parity game is algorithmically challenging, it is known to be in $\text{UP} \cap \text{co-UP}$ [13]. However, the 2-EXPTIME bound presented in [19] for LTL synthesis holds since the zero-sum parity game obtained is carefully constructed and has a polynomial number of priorities and since the procedure for parity games is exponential in the priorities but only polynomial in the number of states of the arena.

Energy-parity games An energy-parity game is a zero-sum game $\langle \mathcal{G}, \text{Cost}, \text{Obj} \rangle$, where $\text{Obj} = \text{Feas}(\mathcal{G}) \cap \text{Parity}$. Introduced in [6], these games enjoy the following property [7]:

Proposition 9. *Let $\langle \mathcal{G}, \text{Cost}, \text{Obj} \rangle$ be an energy-parity game, if there exists a winning strategy from some state s , then there exists a strategy with memory at most $|\text{St}| \cdot |\text{Pr}| \cdot W$ from s .*

Prop. 9 teaches us that it is sufficient to keep track of cost accumulation up to $|\text{St}| \cdot |\text{Pr}| \cdot W$ along any play. Note that this bound is exponential since W is encoded in binary. Computing the set $\text{Win}[\text{Obj}]$ in those games is an exponential task [7], i.e., computing the set $\text{Win}[\text{Obj}]$ is in $\text{NP} \cap \text{co-NP}$. However, in case $\text{St} = \text{St}_0$ (i.e., one player game), computing such strategy can be done in polynomial time. This is also presented in [7].

Remark 10. *If we consider variants where the cost is bounded from above, we can hope for improvement in the complexity bounds.*

3 Solving Quantitative Rational Synthesis

Careless cooperative case In the careless cooperative case, we reduce the synthesis problem to one-player energy and parity games. We show that a solution exists if and only if a strategy exists in the obtained one-player game.

Fix an LTL formula ϕ over St and call $\mathcal{A}_\phi = \{Q, q_0, \Sigma, \Delta, F\}$ the determinist parity automaton that corresponds to the formula ϕ . The alphabet of \mathcal{A}_ϕ is $\Sigma = \text{St}$ and transition function is $\Delta : Q \times \text{St} \rightarrow Q$. Then the one-player energy parity game $\mathcal{G}[\text{Parity}]$, is obtained as follows: The set of states $\text{St}_{\text{Parity}}$ of $\mathcal{G}[\text{Parity}]$ are the result of the product of state in \mathcal{A}_ϕ with the states in the arena \mathcal{G} i.e. $\text{St}_{\text{Parity}} = Q \times \text{St}$. The edges in this game are defined by the relation

$$\text{Edge}_{\text{Parity}} = \{((q, s), (r, t)) \in (\text{St}_{\text{Parity}})^2 \mid ((s, t) \in \text{Edge}) \wedge (\Delta(q, s) = r)\} .$$

The cost function in $\mathcal{G}[\text{Parity}]$ is defined as

$$\text{Cost}_{\text{Parity}}((q, s), (r, t)) = c \text{ if } ((q, s), (r, t)) \in \text{Edge}_{\text{Parity}}, \text{ and } \text{Cost}(s, t) = c .$$

The priority function in $\mathcal{G}[\text{Parity}]$ is defined by $\text{Pr}((q, s)) = 0$ if $q \in F$ and $\text{Pr}((q, s)) = 1$ otherwise.

Now we fix the LTL formula ϕ , first consider the following formula, $\Phi_{0\text{-NE}} \equiv \bigwedge_{i=1}^{|\text{Agt}|} \neg \phi_i \rightarrow \square \neg \text{Win}[\text{Spec}_i]$ then the automaton \mathcal{A} is obtained for the formula $\phi \equiv \phi_0 \wedge \Phi_{0\text{-NE}}$.

Theorem 11. *Finding a careless solution for Problem 3 is 2-EXPTIME-complete.*

Careful cooperative case In the careful case, we adapt the solution to the purely qualitative case proposed in [9]. We reduce the problem to a one-player parity game, where the objective is a well-chosen LTL formula that encodes the requirements. In the careful case, however, the construction of the parity game is more involved. We propose a new construction that allows us to keep the energy level under check. In both semantic, we capitalize on the existence of resource bounds [7]: if a solution exists, we know that the energy level must stay within these bounds. Hence, even though the space of the possible configurations (state/energy level) is infinite, we can limit our attention to configurations with a bounded energy level.

Non-cooperative setting In the non cooperative setting, one cannot rely on the existence of a resource bound, this is somehow related to the fact that agents can use infinite memory strategy. However in the careless semantics, we show that the rational synthesis problem reduces to a two zero-sum game where player 1 has to ensure that the energy level is non negative along any run that satisfies the parity condition. To the best of our knowledge, such a winning condition has never been addressed in the literature. The case of careful agents is still an ongoing work.

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