A Resource-Sensitive Account of the Use of Artifacts

(Extended Abstract)

Daniele Porello
ISTC-CNR, Trento, Italy
daniele.porello@loa.istc.cnr.it

Nicolas Troquard
ISTC-CNR, Trento, Italy
nicolas.troquard@loa.istc.cnr.it

ABSTRACT

The aim of this abstract is to introduce a formal framework enabling to reason about resource-sensitive uses of artifacts. To achieve this, we integrate (non-normal) modalities into Intuitionistic Linear Logic. The function of an artifact is a (resource-sensitive) linear implication and we interpret each modality as an agent's bringing about of resources.

Categories and Subject Descriptors

I.2.4 [**Artificial Intelligence**]: Knowledge Representation Formalisms and Methods

General Terms

Theory

Keywords

Artifacts, Functions, Linear Logic, Modalities, Proof Theory

1. PROPOSAL DESCRIPTION

Artifacts are special kind of objects that are characterized by the fact that they are designed by some agent in order to achieve a goal in a particular environment. An important aspect of the modelisation of artifacts is their interaction with the environment and with the agents that *use* the artifact to achieve a specific goal [5]. Briefly, we can view an artifact as a tool that in presence of a number of preconditions $c_1, \dots, c_n$ produces outcomes $o_1, \dots, o_n$. In this work, we want to represent the function of artifacts by means of logical formulas and to view the correct behavior of an artifact by means of a form of reasoning. When reasoning about artifacts and their outcomes, we need to be careful in making all the conditions of use of the artifact explicit, otherwise we end up facing the following unintuitive cases. Imagine we represent the behavior of a screwdriver as a formula that states that if there is a screw s, then we can tighten it t, namely we simply describe the behavior of the artifact as a material implication $s \to t$. In classical logic, we can infer that by means of a single screw driver we can tighten two screws: $s, s, s \to t \vdash t \wedge t$. Thus, without specifying all the relevant constraints on the environment (e.g. that a screwdriver can handle one screw at the time) we end up with unintuitive results. Moreover, we need to specify the relationship between the artifact and the agents: for example, there are artifacts that can be used by one agent at the time. Since a crucial point in modeling artifacts is their interaction with the environment, either we carefully list all the relevant conditions, or we need to change the logical framework that we use to represent the artifact's behavior. In this paper, we propose to pursue this second strategy. Our motivation is that, instead of specifying for each artifact the precondition of its application (e.g. that there is only one screw that a screw driver is supposed to operate on), the logical language that encodes the behavior of the artifact already takes care of preventing unintuitive outcomes. We shall represent artifacts and their functions by means of resource-sensitive logics. Moreover, we shall add a modality to this logic to specify which agent has control over which resource. The logic that we are going to use is a substructural logic, in particular we use a fragment of linear logic, namely intuitionistic multiplicative linear logic (IMLL) [3, 7]. IMLL has several applications as a logic for representing computations [3]. We shall extend IMLL by adding modalities that specify which agent has control over resources. Related work on modalities for IMLL has been developed for example in [1, 6] . The main novelty of our approach is that we use neighborhood semantics in order to define non-normal modalities that are required to model agents' control over resources.

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Language and sequent calculus IMLL. The language of IMLL L_{IMLL} is defined by the BNF $A ::= \mathbf{1} \mid p \mid A \otimes A \mid A \multimap A$, where $p \in \mathit{Atom}$. The resource-sensitive flavor of IMLL is due to the lack of structural rules in the sequent calculus, namely IMLL rejects the global validity of weakening (that amounts to a monotonicity of the entailment) and contraction, that are responsible for example of tautology such as $A \to A \wedge A$ in classical logic; the counterpart in IMLL $A \multimap A \otimes A$ is no longer valid. Exchange still holds, thus contexts of formulas are multisets. The proof-search complexity of IMLL NP-complete [3].

Models of IMLL. We introduce a Kripke-like model for IMLL that is basically due to Urquhart [8]. A *Kripke resource frame* is a structure $\mathcal{M} = (M, e, \circ, \geq)$, where $(M, e, \circ)$ is a commutative monoid with neutral element e, and $\geq$ is a pre-order on M. The frame has to satisfy *bifunctoriality*: if $m \geq n$, and $m' \geq n'$ then $m \circ m' \geq n \circ n'$. The semantics of IMLL is defined as follows. A valuation on atoms $V : \mathit{Atom} \to \mathcal{P}(M)$ has to satisfy the following *heredity* condition: if $m \in V(p)$ and $n \geq m$ then $n \in V(p)$. V

$$\frac{}{A \vdash A}\text{ax} \qquad \frac{\Gamma, A \vdash C \qquad \Gamma' \vdash A}{\Gamma, \Gamma' \vdash C}\text{cut}$$

$$\frac{\Gamma, A, B \vdash C}{\Gamma, A \otimes B \vdash C}\otimes\text{L} \qquad \frac{\Gamma \vdash A \qquad \Gamma' \vdash B}{\Gamma, \Gamma' \vdash A \otimes B}\otimes\text{R}$$

$$\frac{\Gamma \vdash A \qquad \Gamma', B \vdash C}{\Gamma', \Gamma, A \multimap B \vdash C}\multimap\text{L} \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}\multimap\text{R}$$

$$\frac{\Gamma \vdash C}{\Gamma, \mathbf{1} \vdash C}\mathbf{1}\text{L} \qquad \frac{}{\vdash \mathbf{1}}\mathbf{1}\text{R} \qquad \frac{\Gamma, A, B \vdash C}{\Gamma, B, A \vdash C}\text{Exchange L}$$

is extended to the language of IMLL as follows. Note that heredity holds for every formula.

$m \models p$ iff $m \in V(p)$.

$m \models \mathbf{1}$ iff $m \geq e$.

$m \models A \otimes B$ iff there exist m_1 and m_2 such that $m \geq m_1 \circ m_2$ and $m_1 \models A$ and $m_2 \models B$.

$m \models A \multimap B$ iff for all $n \in M$, if $n \models A$, then $n \circ m \models B$.

A formula A is *true* in a model $\mathcal{M}$ if $e \models A$. A formula A is *valid* in Kripke resource frames iff it is true in every model. Moreover, a sequent $A_1, \ldots, A_n \vdash B$ is *valid* in a Kripke resource frame iff the formula $A_1 \otimes \ldots \otimes A_n \multimap B$ is valid. The calculus of IMLL presented above is sound and complete wrt. the class of Kripke resource models [8].

Modalities for IMLL. For our application, we integrate a *bringing-it-about* modality [2, 4]. For each agent a in a set $\mathcal{A}$, we define a modality E_a, and $E_a A$ specifies that agent $a \in \mathcal{A}$ brings about the state of affairs A. We define a neighborhood semantics on top of the Kripke resource frame. A neighborhood function is a mapping $N_a : M \to \mathcal{P}(\mathcal{P}(M))$ that associates a world m with a set of sets of worlds. Denote $||A||$ the extension of A, i.e. the set of worlds in which A holds.

$$m \models E_a A \text{ iff } ||A|| \in N_a(m) \qquad (1)$$

To conserve heredity for arbitrary formulas and soundness of existing rules, we need: if $X \in N_a(m)$ and $m' \geq m$ then $X \in N_a(m')$.

We want our modalities to satisfy the principle T: $E_a A \multimap A$. In our interpretation, it means that if an agent brings about A, then A can be used in the environment. The general neighborhood semantics does not make T true. We prove that the following condition does the job.

$$\text{if } X \in N_a(w) \text{ then } w \in X \qquad (2)$$

We prove that if (2) holds, then, for every model $\mathcal{M}$, we have that $e \models E_a A \multimap A$. By definition of $\multimap$, $e \models E_a A \multimap A$ iff for all $m \in M$, if $m \models E_a A$, then $m \models A$. By (1), if $m \models E_a A$, then $||A|| \in N_a(m)$. That entails, by (2), that $m \in ||A||$, thus $m \models A$.

We enrich the sequent calculus of IMLL by adding the following rules.

$$\frac{A \vdash B \qquad B \vdash A}{E_a A \vdash E_a B} E_a(\text{re}) \qquad \frac{\Gamma, A \vdash B}{\Gamma, E_a A \vdash B} E_a(\text{refl})$$

The sequent E_a(re) ensures that E_a is a classical modality. The sequent E_a(refl) has the same effect on the logic than the axiom T.

The use of artifacts. We represent the behavior of an artifact as a formula $P \multimap O$ where P is a tensor of preconditions and O is a tensor of outcomes. We model the way in which the artifact $P \multimap O$ provides the expected outcome O, by means of proof search in the enriched calculus. Take a very simple example. We can represent the behavior of a screwdriver as an implication that states that if there is a screw (s) and some agent can apply the right force (f), then the screw is tight (t): $s \otimes f \multimap t$. Suppose the environment provides s and an agent i is providing the right force, that is $E_i f$, we can show that the goal t can be achieved by means of the following proof.

$$\dfrac{\dfrac{s \vdash s \qquad \dfrac{f \vdash f}{E_i f \vdash f} E_a\ (\text{refl})}{s, E_i f \vdash s \otimes f} \otimes \text{R} \qquad t \vdash t}{s, E_i f, s \otimes f \multimap t \vdash t} \multimap\text{L}$$

Our calculus is resource sensitive, thus, as expected, we cannot infer for example that two agents can use the same screwdriver at the same time to tighten two screws: $s, s, E_i f, E_j f, s \otimes f \multimap t \nvdash t \otimes t$. By exploiting this methodology we can represent more complex artifacts that for example may require a number of agents in order to achieve the goal or we can easily combine artifacts by using the connective of IMLL.

Conclusions and ongoing work. We integrated a minimal modal logic into IMLL, and interpreted the modalities as agents' bringing about states of affairs. Ongoing work concerns the theoretical aspects of non-normal substructural modal logic, and its application to represent and reason about resource-sensitive use of artifacts. It can be shown that the calculus of IMLL extended with rule E_a(re) and rule E_a(refl) is sound a complete wrt. to the class of Kripke resource models extended with N_a.

In the future, we plan to propose complete resource-sensitive versions of bringing-it-about logic and of coalition logic.

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