# Towards a logic of agency and actions with duration

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**Abstract.** As far as we know, there is no multi-agent system allowing to talk both about choices of agents or groups of agents, strategies, and about sufficiently rich actions. This paper aims at offering a path towards a new more expressive logical framework by mixing a STIT-like logic of agency with a PDL-like logic of action. We present the syntax and ontological motivations, and we highlight the expressivity of the resulting framework on an example.

## 1 Introduction

Many domains, e.g., agent interaction or social law modeling, require a good framework for time, agency and action. Time is the basis to express dynamic properties and indeterminacy of the future, agency deals with what agents can bring about and actions are the various ways to bring about some state of affairs. As far as we know, there is no multi-agent system allowing to represent these three domains with sufficient expressivity. In particular, we intend to cover actions that have a duration, and that can be categorized on the basis of properties such as expected effects, temporal or participant structure.

Some existing logics answer to some extent such needs. Concerning pure action, the well-known Propositional Dynamic Logic (PDL) is a natural candidate. Nevertheless, it is not suitable neither for group action nor for individual and group agency. The logic of "Seeing To It That" (STIT) is a logic of agency embedded in a branching-time framework [1]. This is a logic about choices and strategies for individuals and groups. The core idea of logics of agency is that *acting* is best described by what an agent brings about: at some time, an agent *chooses* to constrain that some proposition is true. However, in some circumstances, not being able to explicitly refer to *actions* remains a weakness. One expresses sentences of the form "Mary sees to it that the coyote is dead" but not "Mary shoots at the coyote" or "Mary poisons the coyote", i.e., the manner of bringing a state of affairs is out of concern. In addition, in STIT, it is generally considered that Mary's acting does not take time: actions cannot be suspended half-way and one cannot express that an action starts while another is going on. This last point has already been overcome in [3] with the operator $istit$, but this logic still doesn't involve actions explicitly.

It appears that we need a richer logical system, for reasoning about time, agency and actions with duration. One research avenue is to capitalize on strength of both PDL and STIT. The aim of this paper is to investigate this avenue, offering an expressive logical framework to support time, agency (for individuals and groups) and actions with duration and other properties, for modeling interactions between agents.

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## 2 Description and justification of the framework

Within the limits of this extended abstract, we give the language of the new logic and describe some elements of its semantics intertwined with ontological justifications. Possible axioms or theorems are proposed in formulas labelled ***(n)***. Models have been fully characterized but axiomatization proper is still work in progress.

**Language.** $\mathcal{A}ct$ is the set of actions, and $\mathcal{A}ct_\lambda := \{\alpha_\lambda \mid \alpha \in \mathcal{A}ct\}$ is the set of continuations of those actions. $\mathcal{A}tm$ is the set of atomic propositions. $\mathcal{A}gt$ is the set of agents. By notational convention, $\alpha \in \mathcal{A}ct$, $\alpha_\lambda \in \mathcal{A}ct_\lambda$, $\beta \in \mathcal{A}ct \cup \mathcal{A}ct_\lambda$, $p \in \mathcal{A}tm$, $a \in \mathcal{A}gt$ and $A \subseteq \mathcal{A}gt$. A formula can have the following syntactic form:

$$\varphi \quad \triangleq \quad \bot \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi\mathcal{S}\varphi \mid \varphi\mathcal{U}\varphi \mid \Box\varphi \mid [\beta : a]\varphi \mid [\overline{\alpha : a}]\varphi \mid Stit_A\varphi$$

A model is a tuple $\mathcal{M} = \langle W, <, R_\Box, R_{ACT}, agent, v\rangle$, where $W$ is a set of indexes, partially ordered by the strict temporal precedence $<$. Non comparable indexes are grouped into moments by the equivalence relation $R_\Box$. $R_{ACT}(\beta)$ is a function associating an index $w$ to the index where the performance at $w$ of $\beta$ ends, $agent$ is a function associating to each action its agent, $v$ is the valuation function.

As in STIT, $\mathcal{S}$ and $\mathcal{U}$ are the standard since and until temporal operators. Future and past operators are defined as usual: $\mathbf{F}\varphi \equiv \top\mathcal{U}\varphi$ and $\mathbf{P}\varphi \equiv \top\mathcal{S}\varphi$. $\Box\varphi$ stands for historical necessity of $\varphi$ ($\varphi$ is true at every index of the moment). $\Box$ is an $S5$ modal necessity **(1)**. Possible readings of $Stit_A\varphi$ are "the group $A$ sees to it that $\varphi$" or "the group $A$'s current choice ensures $\varphi$ whatever other agents do".

As in PDL, $[\beta : a]$ means that $a$ starts performing action $\beta$ and that $\varphi$ holds afterwards. $[\overline{\alpha : a}]\varphi$ means that $\alpha$ has just finished and that $\varphi$ was true before. By tradition, $\Diamond$ will be used as abbreviation for $\neg\Box\neg$ and similarly for $\langle\beta : a\rangle$ and $[\beta : a]$.

We will illustrate the logic by the example on Figure 1.

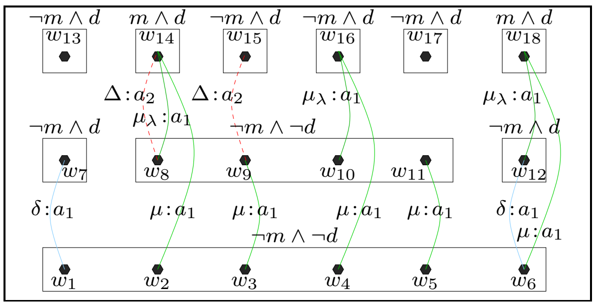


**Figure 1.** The two cooks. $a_1$ and $a_2$ have to prepare a meal. $m$ stands for "the main course is done" and $d$ for "the dessert is done". $\delta$ and $\Delta$ are actions of (resp.) $a_1$ and $a_2$ cooking the dessert, while $\mu$ is the action of $a_1$ cooking the main course. Boxes are moments containing indexes.

**Time.** As in STIT, our logic assumes a branching time on moments, linear in the past: at each moment, an agent can make different choices, that is, decide to execute different actions bringing about different futures. Maximal linear sets of moments are called histories; indexes can be seen as moment-history pairs. Models do not constrain all moments to be temporally comparable; an extension could be to do so, adding coincidence of moments through the notion of *instant* as in [1].

**Actions.** All actions take time ($[\beta : a]\varphi \rightarrow \mathbf{F}\varphi$ **(2)**). Actions in the present logic are operators thus not properly speaking "events performed by an agent" since events, when they are acknowledged as citizens of the world, are conceived of as concrete individuals, uniquely situated in time and space [2]. These operators correspond to *types*, not tokens, as a given action may occur repeatedly. They are though of a very restricted sort of types. The agent, as well as all other participants, are fixed. The only remaining parameter is time.

Actions correspond to achievements and accomplishments [5], thus two occurrences of the same action cannot overlap ($[\alpha : a]\mathbf{P}[\overline{\alpha{:}a}]\varphi \rightarrow \mathbf{P}[\overline{\alpha{:}a}]\varphi$ **(3)** and $\langle\alpha{:}a\rangle\top \wedge \mathbf{F}[\alpha{:}a]\varphi \rightarrow [\alpha{:}a]\mathbf{F}[\alpha{:}a]\varphi$ **(4)** are expected to be valid). Each occurrence runs linearly ($\langle\beta{:}a\rangle\varphi \leftrightarrow [\beta{:}a]\varphi$ **(5)**, $\langle\overline{\alpha{:}a}\rangle\varphi \leftrightarrow [\overline{\alpha{:}a}]\varphi$ **(6)**, $[\alpha{:}a][\overline{\alpha{:}a}]\varphi \rightarrow \varphi$ **(7)** and $\varphi \wedge \langle\alpha{:}a\rangle\top \rightarrow [\alpha{:}a][\overline{\alpha{:}a}]\varphi$ **(8)** are assumed to be valid).

At a same index, more than one action can be performed, by the same or another agent. In the above example, at $w_6$, $a_1$ performs $\mu$ and $\delta$ ($w_6 \models \langle\mu{:}a_1\rangle\top \wedge \langle\delta{:}a_1\rangle\top$).

An action is simply executed or not at an index, but it can unfold into different courses at different indexes of a same moment: in agreement with the STIT approach, actions are not deterministic. In particular, the duration of an action may be left unspecified. That is, not only different occurrences may have different lengths, but the possibly different courses of the same action occurrence may have different lengths on different histories. Action duration can for instance be influenced by the availability of resources. It is also influenced by the fact that actions may be suspended before completion, for reasons external or internal to the agent [3]. Since actions may abort, starting an action does not imply obtaining some expected result: $[\alpha : a]\varphi \rightarrow \Box[\alpha : a]\varphi$ is not valid. This means that in our approach, actions are not simply characterized by preconditions and results; we rather focus on the decision of the agent to perform an event of some sort.

**Continuation of an action, completed actions.** Assuming that actions have a duration and can abort before completion allows to assume that the agent has control over the execution of an action. At each moment during the execution, the agent can decide to keep on performing it or not. On the other hand, in a STIT framework with several agents, agents share the set of indexes, and as a result, whenever an agent makes a choice, all other agents too. This appears too demanding, as simply continuing what has been initiated before is not really a new choice. For both these reasons, we introduce particular actions representing the *continuation of an action*. Introducing in an explicit manner continuations of an action is actually a good way to formalize the notion of *control* on the action [4].We follow Searle in holding that actions end when the bodily movement is finished, i.e., all actions are totally under control and thus continued up to their end ($\langle\alpha : a\rangle\top \rightarrow \langle\alpha_\lambda : a\rangle\top\mathcal{U}\langle\overline{\alpha{:}a}\rangle\top$ **(9)** is assumed to be valid). Of course, if a continuation is available, it means that the corresponding action has started before ($[\alpha_\lambda : a]\varphi \rightarrow \mathbf{P}[\alpha : a]\varphi$ **(10)**). Continuations of a given occurrence of an action do not have a unique starting point, as they are repeated till the end of the action, but they all run till the same ending point (cf. Formula 9).

Since actions can abort, when an action ends, it is not necessarily completed. We thus introduce propositions $comp(\alpha) \in \mathcal{A}tm$, one for each action $\alpha$, that reads "action $\alpha$ is completed". An action $\alpha$ is completed when it has just ended and no continuation is possible; an action aborts if it ends but some continuation is still possible. In our example, action $\mu$ is aborted in $w_9$ and $w_{11}$ and completed in $w_{14}$, $w_{16}$ and $w_{18}$. This notion of completion may be used to express that completed actions do have specific effects; categories of actions could then be introduced on the basis of effects of completed actions. However, one may also want to allow for completed but failed actions, just as in $w_{16}$, where $m$ doesn't hold.

**Not doing anything.** As observed before, STIT's requiring that if an agent makes a choice at a moment, all other agents too, is too demanding, and needs to be fixed not only for action continuations. In fact, agents may remain simply passive when others really choose to act. To express this, we introduce a set of propositions $\lambda(a) \in \mathcal{A}tm$, one for each agent $a$, that reads "the agent $a$ remains passive". An agent $a$ remains passive when it does not perform an action nor a continuation. In the example, agent $a_2$ remains passive everywhere but $w_8$ and $w_{10}$ ($w_8 \models \neg\lambda(a_2)$).

**Choices and groups agency.** We can analyze the combination of choice and action. In multi-agent systems, and particularly in STIT, an agent's choice is understood as choosing to bring about a state of affairs. In the present work, we handle choice as choosing to perform a set of actions. To deal with agency we still use the $Stit$ operator. $Stit$ is an $S5$ modal operator **(11)**. Moreover, if $A \subseteq B$ then $Stit_A\varphi \rightarrow Stit_B\varphi$ **(12)**. We have the interesting property that if $a$ performs $\alpha$, $a$ sees to it that it performs $\alpha$, which can be stated by $\langle\alpha : a\rangle\top \leftrightarrow Stit_a\langle\alpha : a\rangle\top$. Similarly, it is also true that $\lambda(a) \leftrightarrow Stit_a\lambda(a)$. In the example, at the moment of $w_1$, agent $a_1$ has three choices, corresponding to performing action $\delta$, performing action $\mu$ or performing both ($w_2 \models Stit_{a_1}\langle\mu{:}a_1\rangle\top \wedge \Diamond Stit_{a_1}\neg\langle\mu{:}a_1\rangle\top$).

Concerning cooperation, at $w_8$ none of $a_1$ and $a_2$ can see to it that both the main course and the dessert are cooked ($\neg\Diamond Stit_{a_1}(m \wedge d) \wedge \neg\Diamond Stit_{a_2}(m \wedge d)$) but they can cooperate for that ($w_8 \models Stit_{\{a_1,a_2\}}(m \wedge p)$); it is achieved by means of $a_1$ continuing to perform $\mu$ and $a_2$ executing $\Delta$.

## 3 Conclusion

In this work, we have introduced actions with duration, action continuations, and explicit choices of remaining passive in the language of a STIT-like framework. By doing so, we have also cured an annoying feature of STIT which is that when an agent makes a choice, other agents too. Moreover, choices in STIT are arbitrary partitions of moments; we made the notion of choice clearer by constructing choices over sets of actions.

Besides working out a full axiomatization, extensions include: less restricted types of action, expected results, action temporal structure, composing operators and strategies for individuals and groups.

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